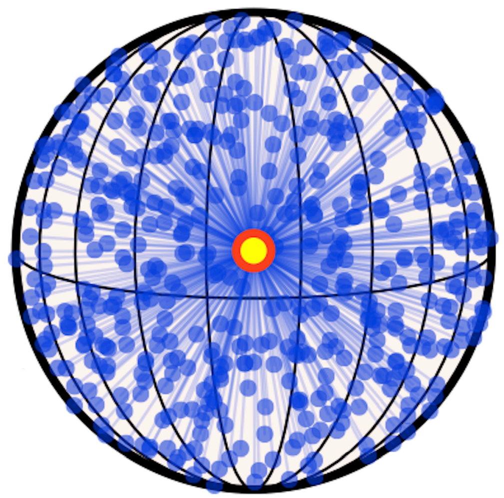
TII, Abu Dhabi, UAE
20 November 2024

Deep Thermalization and Generalized Maximum Entropy Principles

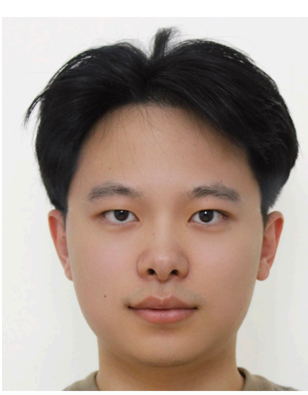


Wen Wei Ho
National University of Singapore (NUS)
Centre for Quantum Technologies (CQT)

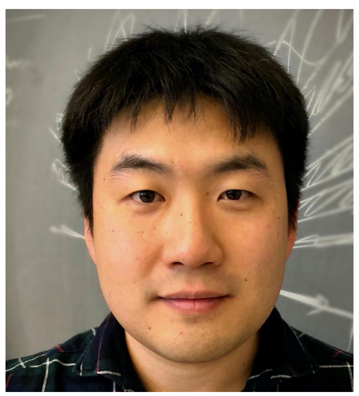
[WWH, S. Choi, PRL 128, 060621 (2022)]
[Ippoliti and WWH, PRX Q 4, 030322 (2023)]
[Ippoliti and WWH, Quantum 6, 886 (2022)]
[Shrotriya and WWH, arXiv:2305.08437]
[Liu, Huang and WWH, arXiv: 2405.05470]
[Chang, Shrotriya, WWH, Ippoliti, 2408.15325]

Harshank Shrotriya
CQT, NUS

Chang Liu
NUS

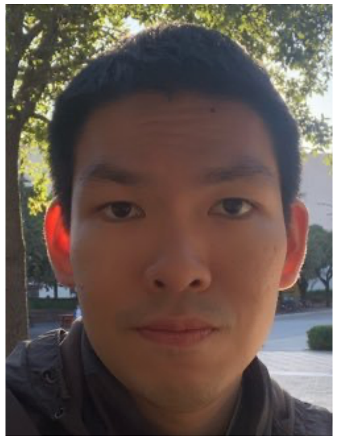


Qi Camm Huang
NUS



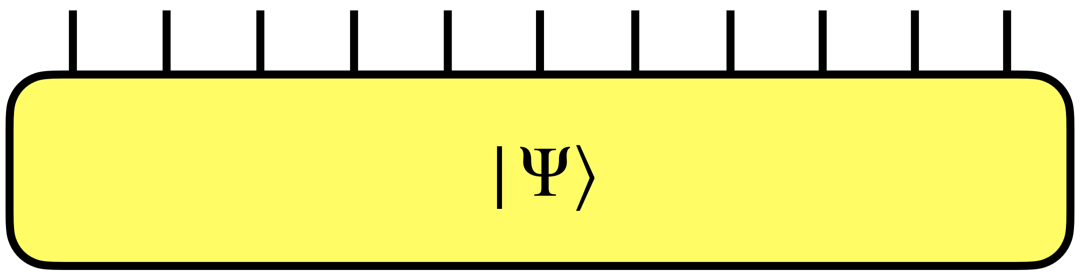
Soonwon Choi
MIT

Matteo Ippoliti
UT Austin



Rui-an Chang
UT Austin

What sorts of universal structures are contained within generic quantum many-body states?



Generic = Highly entangled, containing large amounts of magic

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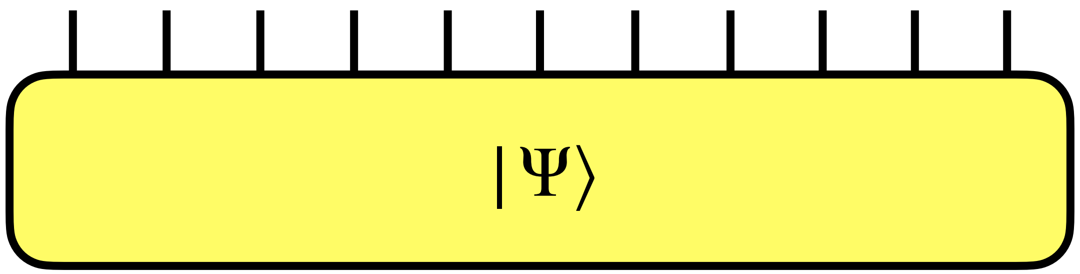

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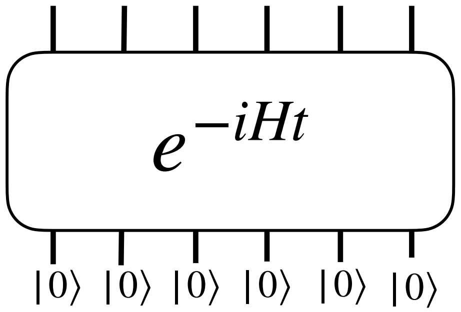
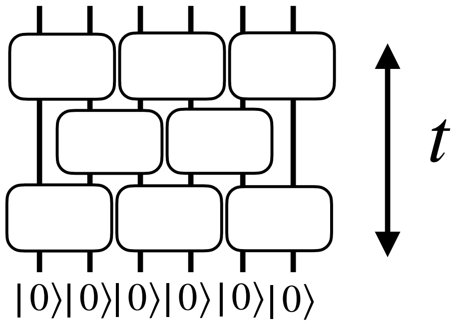
To fix thoughts, consider the nonequilibrium setting of quantum quenches beginning from a simple product state:



Hamiltonian evolution



Circuit evolution

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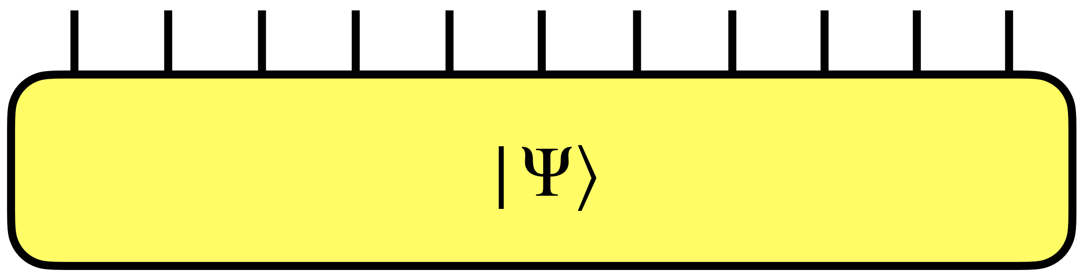

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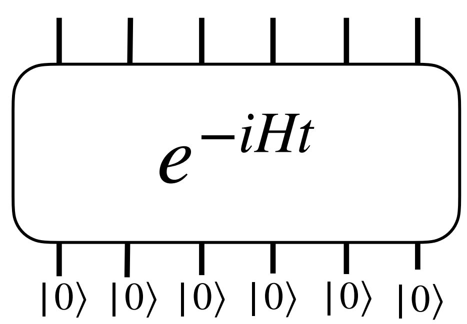

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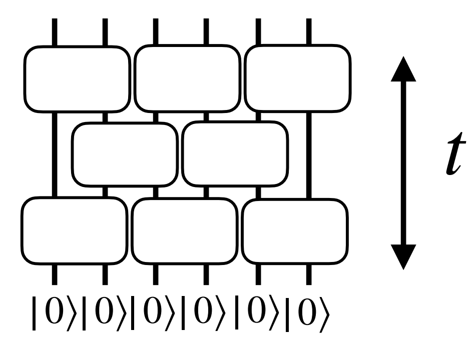
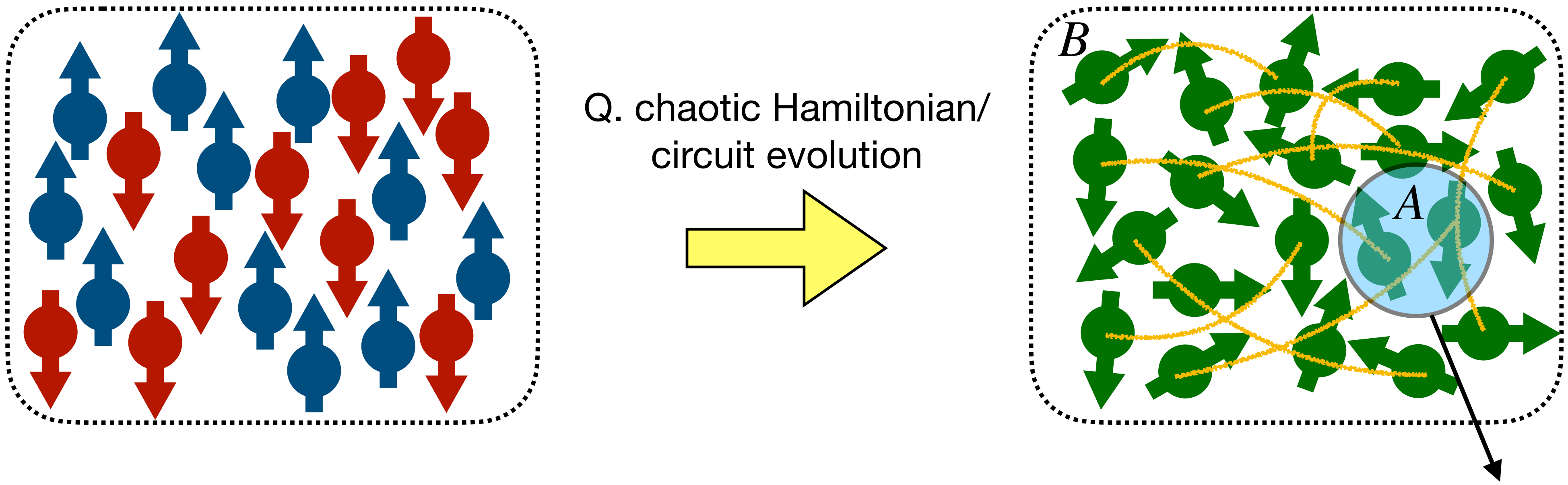


Circuit evolution

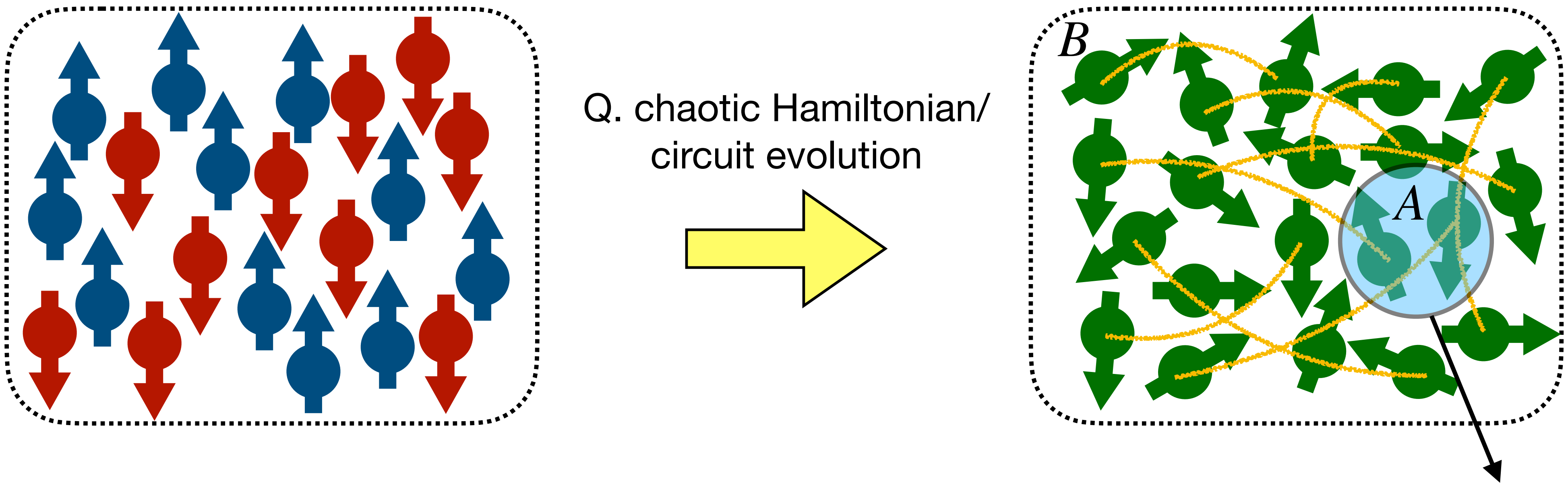
Entanglement and magic are being generated over time; how do they affect the resulting universality (if any), and how quickly is this achieved?

Quantum Thermalization



$$\rho_A(t) = \mathrm{Tr}_B(|\Psi_t\rangle\langle\Psi_t|)$$

Quantum Thermalization



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Universality:

$$\lim_{t\to\infty} \rho_A(t) \to \propto e^{-\beta H_A}$$

(Gibbs state)

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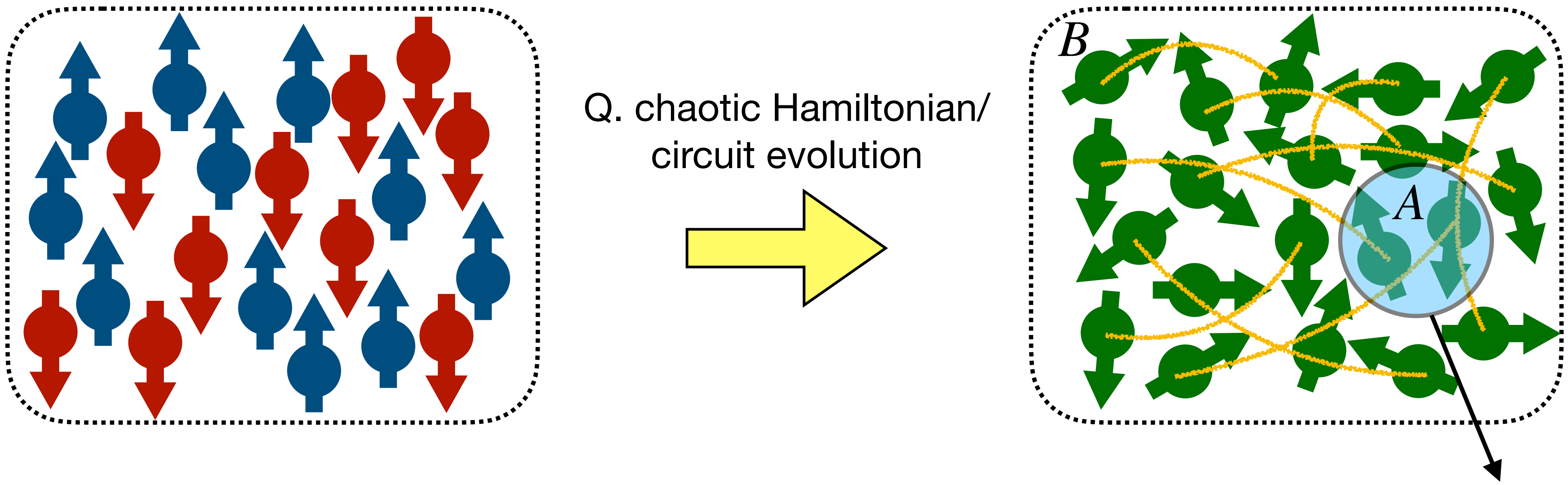

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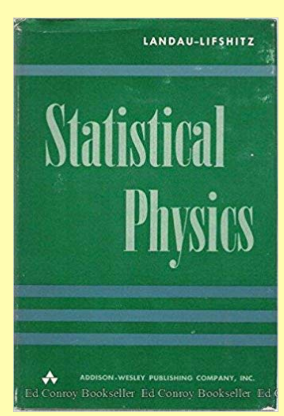
Principle:



2nd law of thermodynamics:

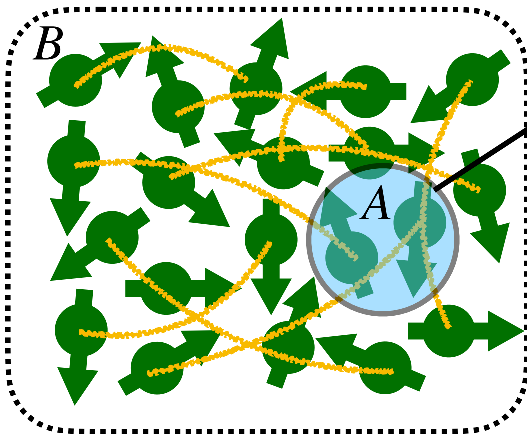
Minimize free energy ↔

Maximize von Neumann entropy $S(\rho_A)$

subject to conservation laws

Beyond quantum thermalization?

Examine assumptions of framework:



$\rho_A = \mathrm{Tr}_B(|\Psi\rangle\langle\Psi|)$ Reduced density matrix

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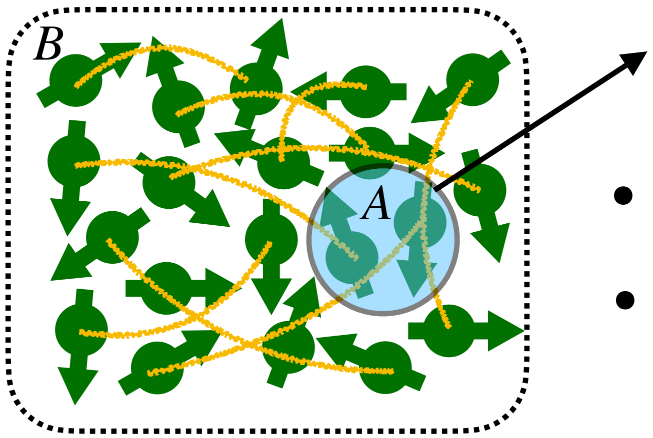

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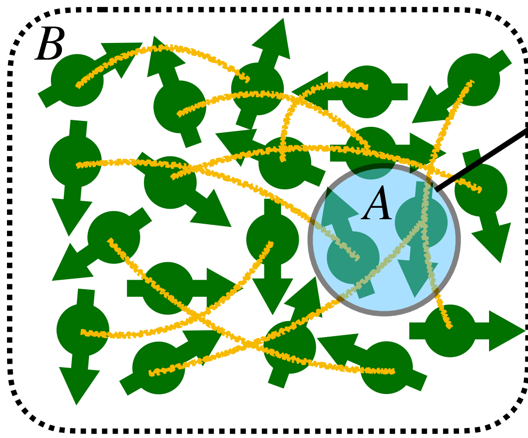

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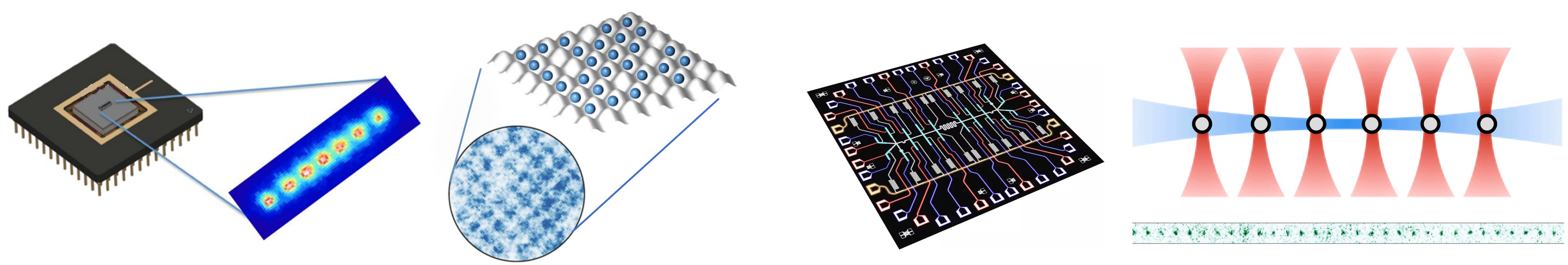
Quantum Simulators allow us to probe beyond this framework:



Trapped ions
[Duke, Innsbruck, Honeywell…]

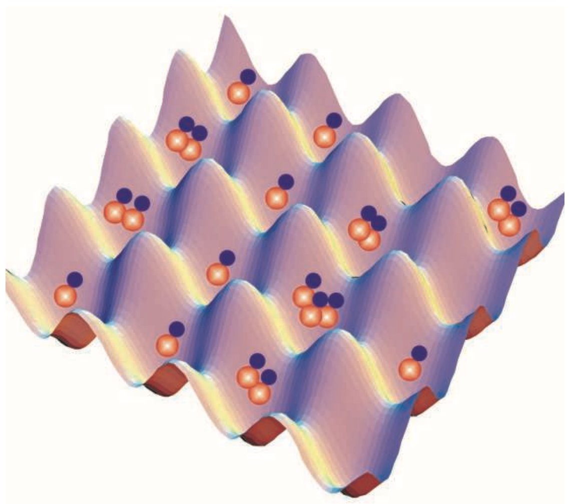
Cold atoms
[Munich, Harvard, MIT…]

Superconducting qubits
[Google, IBM,…]

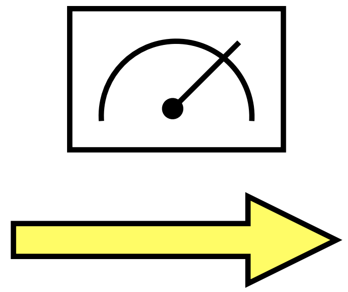
Rydberg atoms
[Harvard, Caltech, Paris,…]

✓ Microscopically-resolved global measurements $\Longrightarrow$
Access beyond local observables!

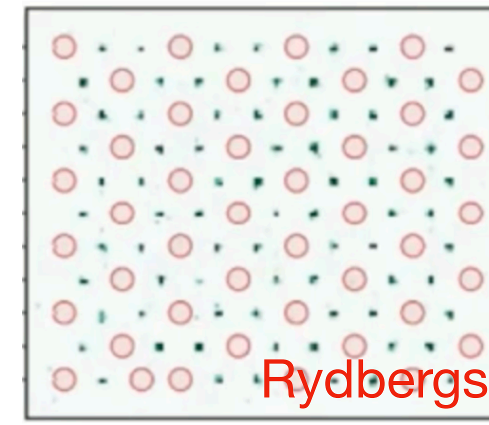
Measurements in quantum simulators



Quantum simulator

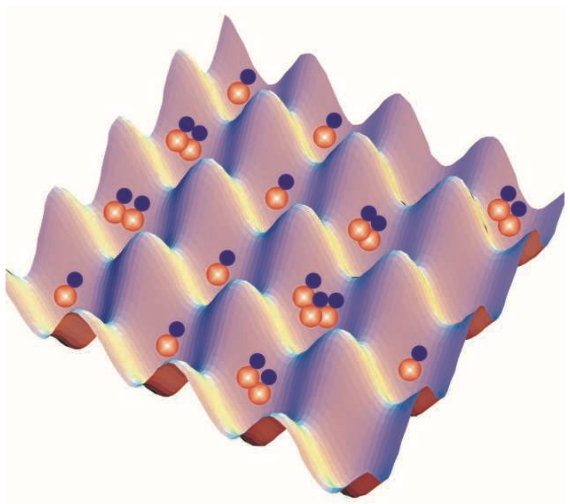


$$\vec{z}^{(1)} = 010110101101001$$
$$\vec{z}^{(2)} = 101010111011011$$
$$\vec{z}^{(3)} = 110110101101001$$
$$\vec{z}^{(4)} = 000101100101110$$
$$\vec{z}^{(5)} = 111010101000010$$
$$\vec{z}^{(6)} = 011110101101001$$
$$\vec{z}^{(7)} = 110101000101011$$



[Lukin group, Harvard]

Measurements in quantum simulators



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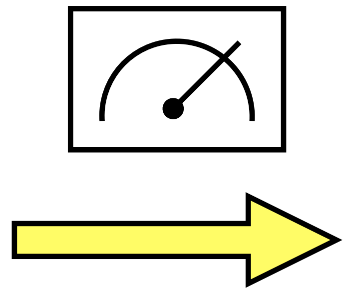
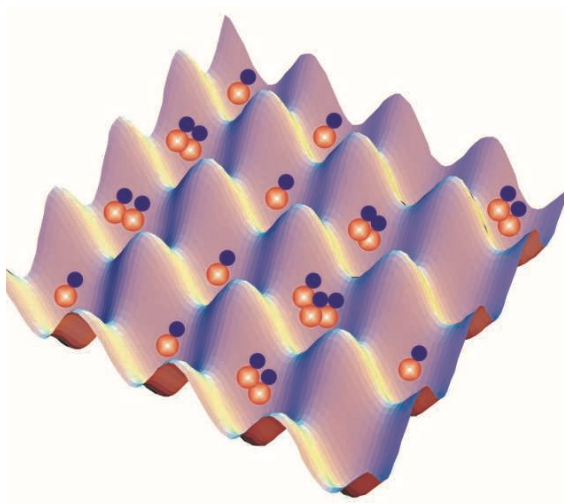


0101101011010 01
1010101110110 11
1101101011010 01
0001011001011 10
1110101010000 10
0111101011010 01
1101010001010 11

A *B*

Measurements in quantum simulators



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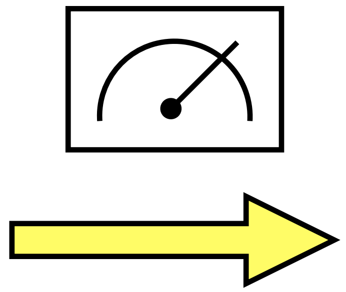
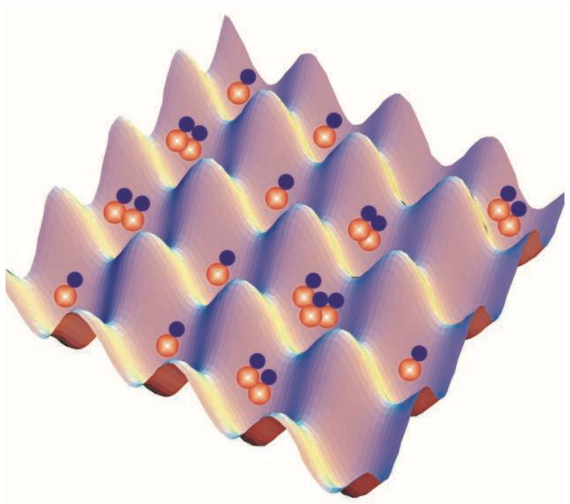


A	B
010	110101101001
101	010111011011
110	110101101001
000	101100101110
111	010101000010
011	110101101001
110	101000101011

$$\langle Z_1 \rangle \approx \frac{1}{M} \sum_i \vec{z}_1^{(i)}$$

Measurements in quantum simulators



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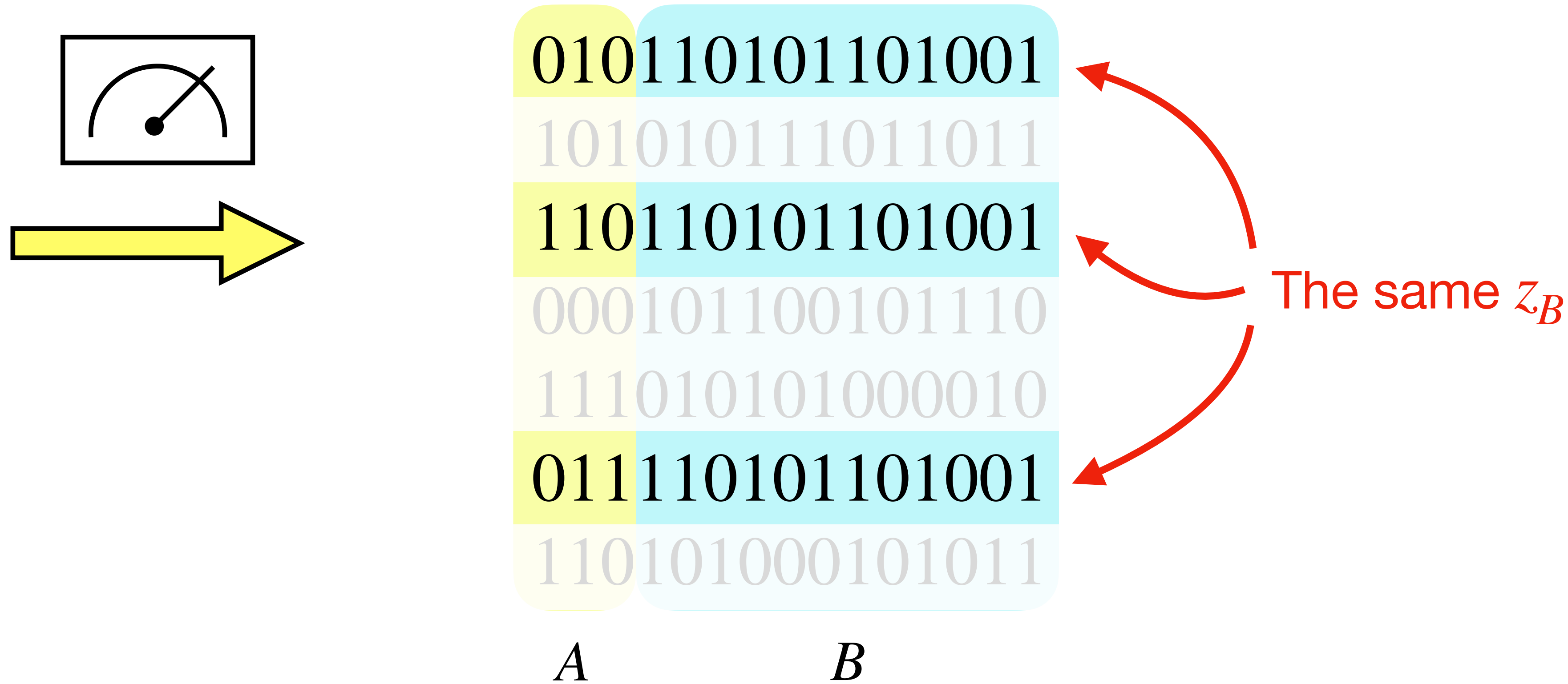
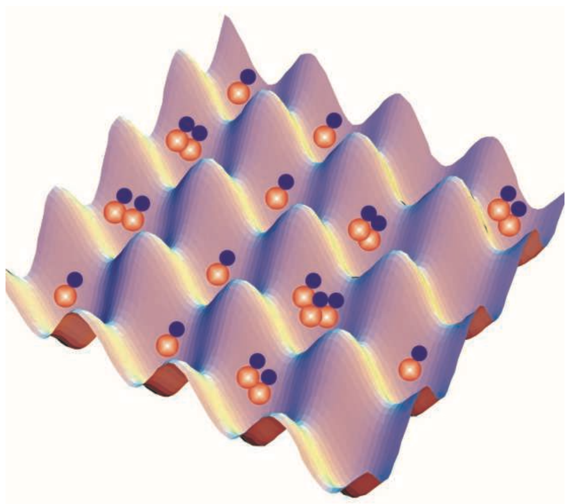


- Access to **conditional** local quantum observables:

$\langle O_A \rangle_{z_B}$:= Expectation value of O_A conditioned upon observing state z_B

Measurements in quantum simulators



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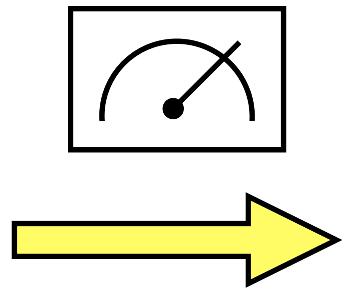

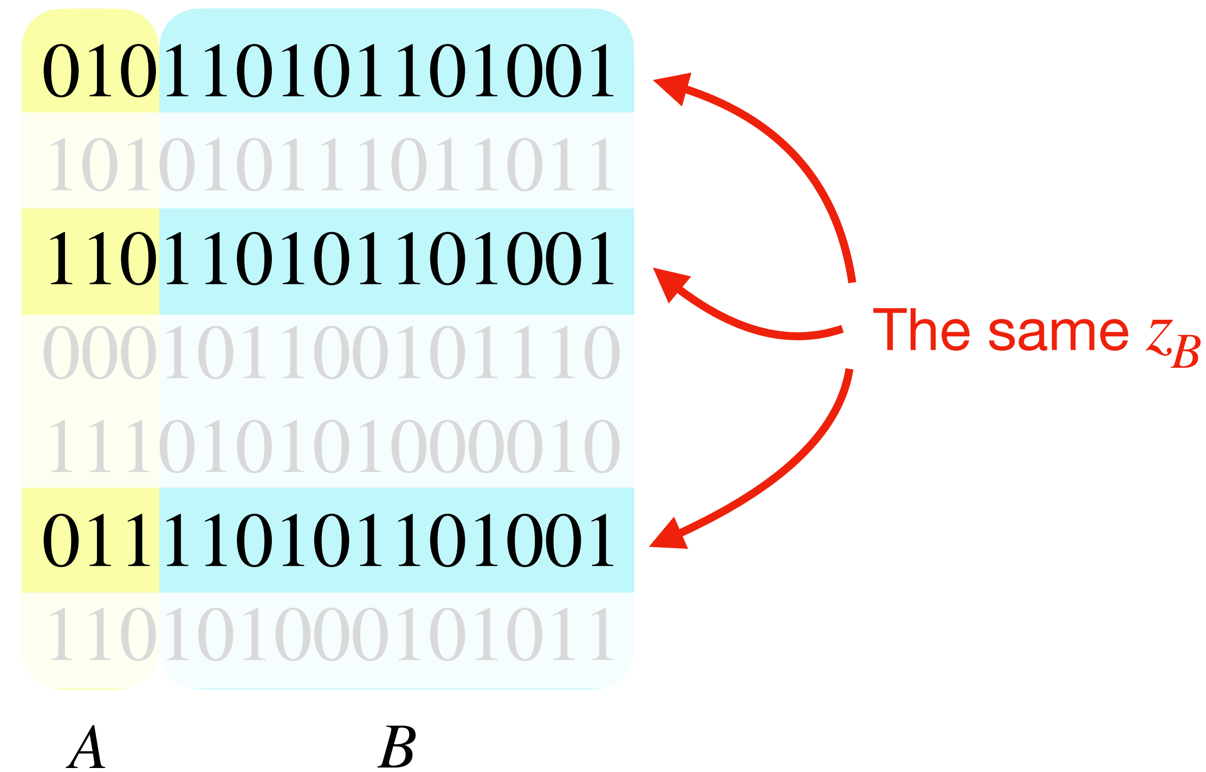

- Access to **conditional** local quantum observables:

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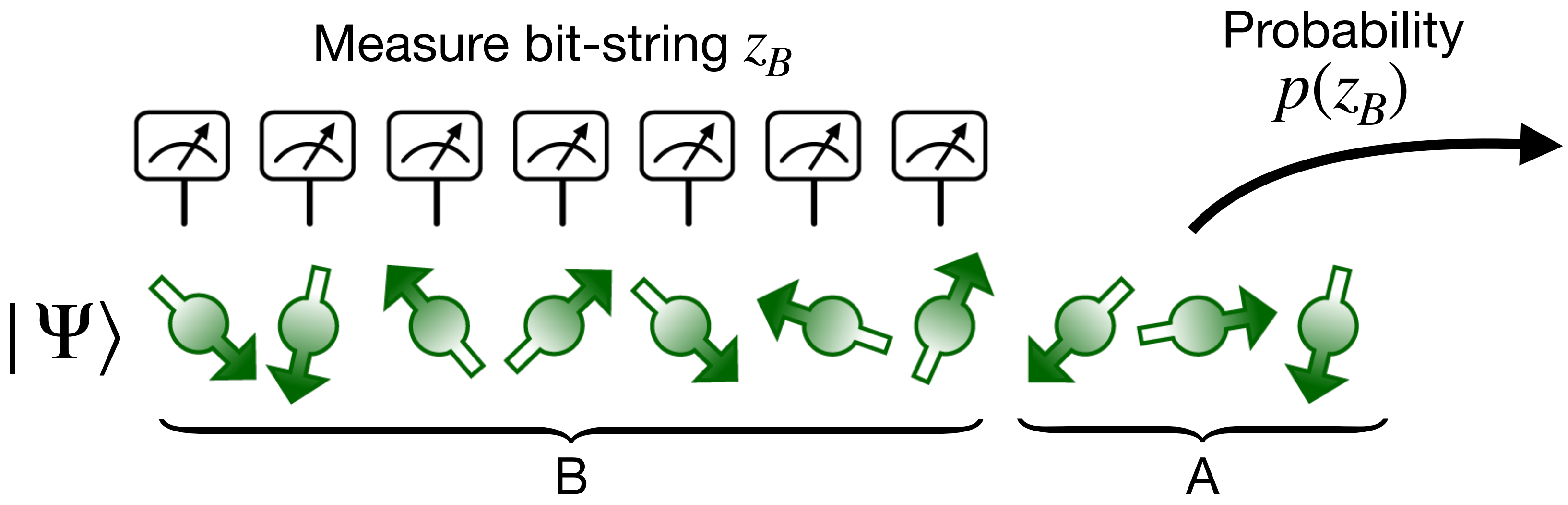
*Hybrid **quantum-classical** observable, beyond the reduced density matrix*

Theoretical framework to capture such quantities?

Projected ensemble

[PRX Quantum 4 (1), 010311 (2023), Nature 613 7944 (2023)]



$$|\psi_A(z_B)\rangle = \frac{(\mathbb{I}_A \otimes \langle z_B|)|\Psi\rangle}{\sqrt{p(z_B)}}$$

$$p(z_B) = \langle \Psi | \mathbb{I}_A \otimes |z_B\rangle\langle z_B| \, |\Psi\rangle$$

Projected ensemble

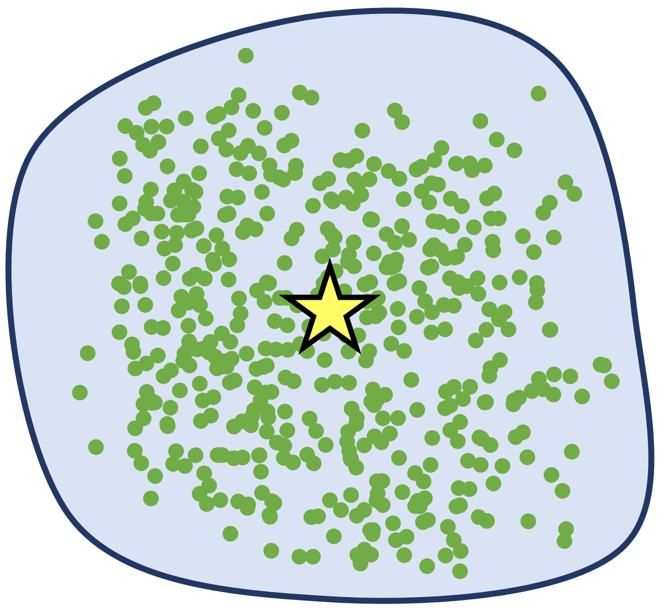
Collection of probabilities + pure quantum states

$$\mathcal{E}_A = \left\{ p(z_B), |\psi_A(z_B)\rangle \right\}$$

equivalently

$$\rho_{QC} = \sum_{z_B} p(z_B) \underbrace{|\psi_A(z_B)\rangle\langle\psi_A(z_B)|}_{\text{Quantum system}} \otimes \underbrace{|z_B\rangle\langle z_B|}_{\text{Classical measurement}}$$

Understanding the projected ensemble



Distribution over Hilbert space

$$\rho_A \mapsto \{p(\psi), |\psi\rangle\}$$

Physically motivated unraveling of density matrix

Fundamental questions

1. Does the projected ensemble tend to a universal limiting distribution in quantum dynamics?

$$\lim_{t\to\infty} \mathcal{E}_A \xrightarrow{?} \mathcal{E}^*_A$$

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This talk:

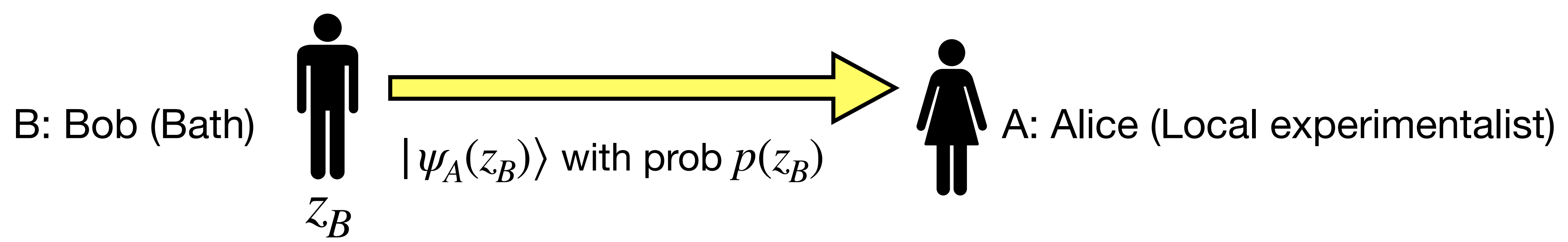
Deep thermalization, symmetries, and quantum information theoretic principles

[Liu, Huang, WWH, arXiv: 2405.05470]
Also [Mark et al, arXiv: 2403.11970]

Projected ensemble as a quantum communications protocol

$$|\Psi\rangle \to \mathcal{E}_A = \{p(z_B), |\psi_A(z_B)\rangle\}$$

- PE can be understood as an encoding of classical data z_B into the quantum state $|\psi_A(z_B)\rangle$



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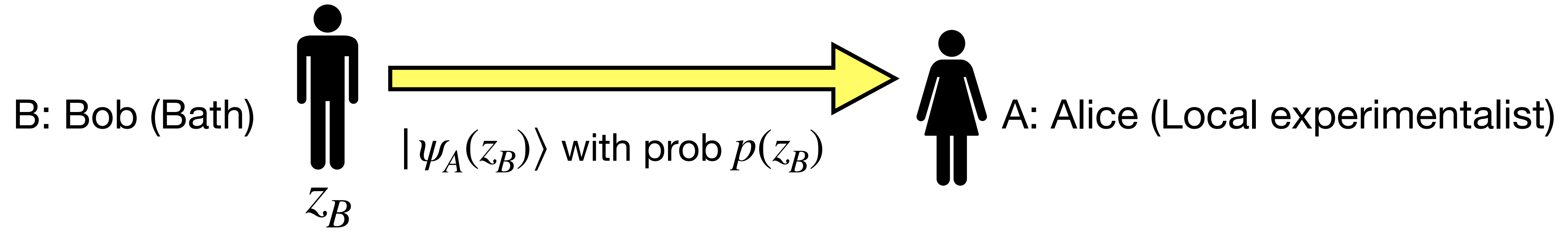
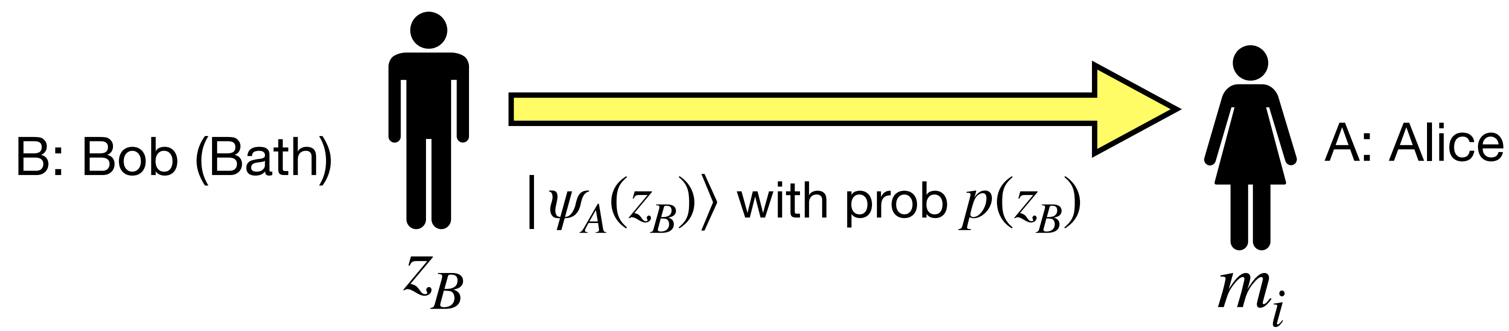


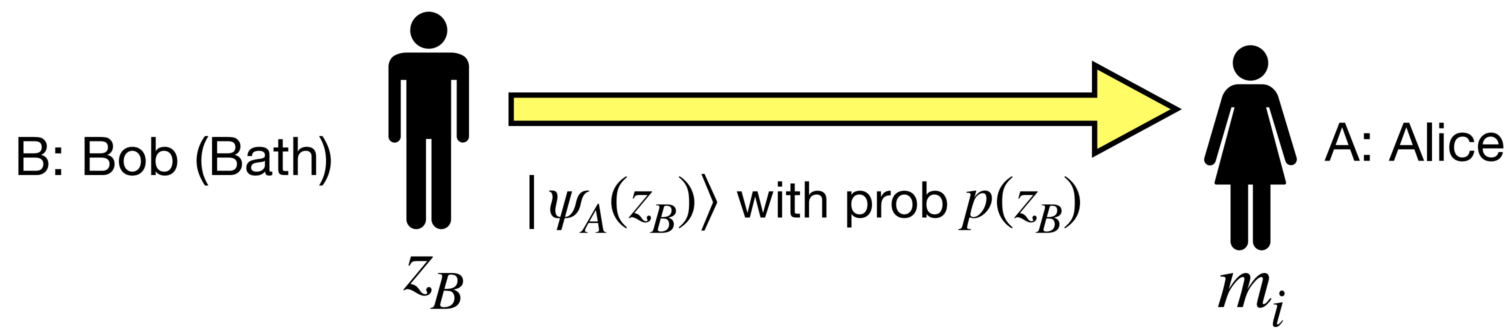
Q: How much information is extractable?

Accessible information



- Alice measures with POVM $\{M_i\}$ and tries to ascertain z_B from measurement outcome m_i

Accessible information



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- Mutual information between messages and measurement outcomes:

$$I(\mathscr{E} : M)$$

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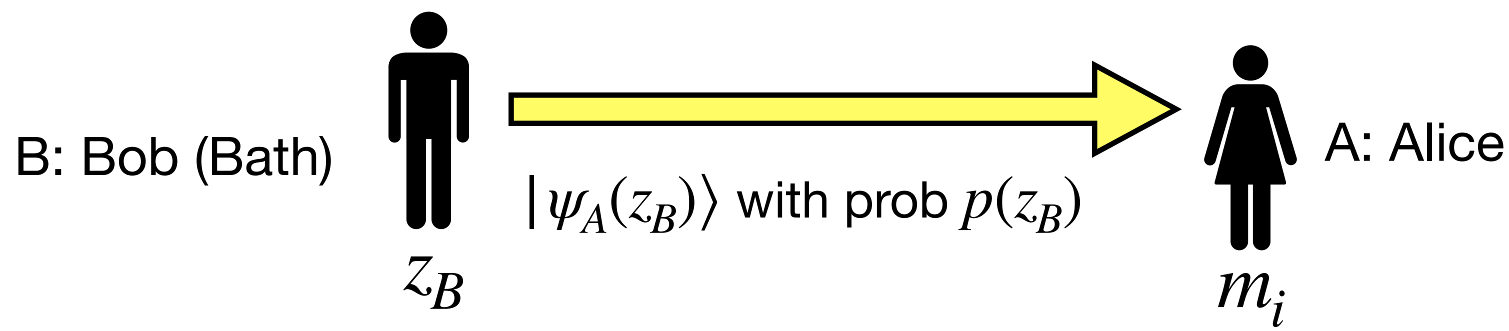

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- Prior to measuring, Alice's prior belief is that z_B is sent with probability $p(z_B)$. Entropy is $H(\mathscr{E}) = -\sum p(z_B)\log p(z_B)$
- After measuring, Alice has posterior belief $p(z_B \,|\, m_i) = p(z_B, m_i)/p(m_i)$. Average entropy is $H(\mathscr{E} \,|\, M) = -\sum_i p(m_i) \sum_{z_B} p(z_B \,|\, m_i)\log p(z_B \,|\, m_i)$
- Averaged information gain is $I(\mathscr{E} : M) = H(\mathscr{E}) - H(\mathscr{E} \,|\, M)$

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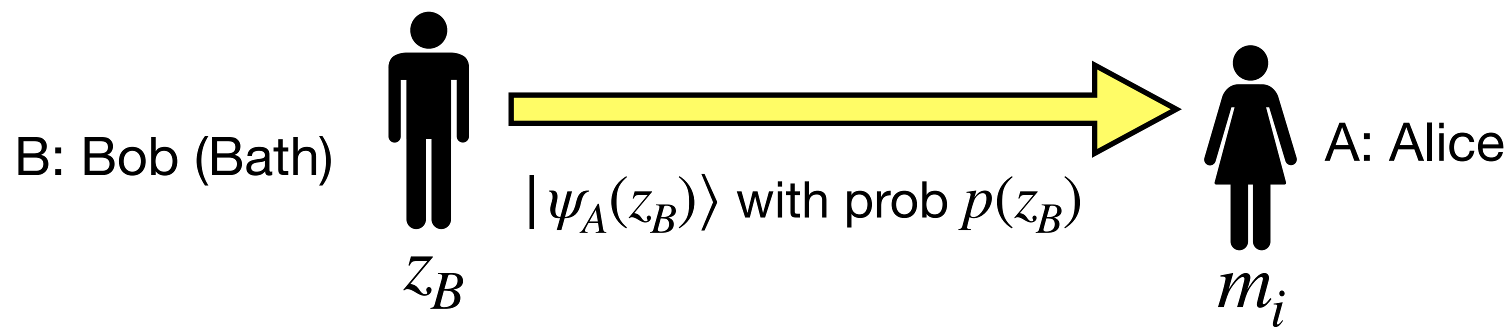

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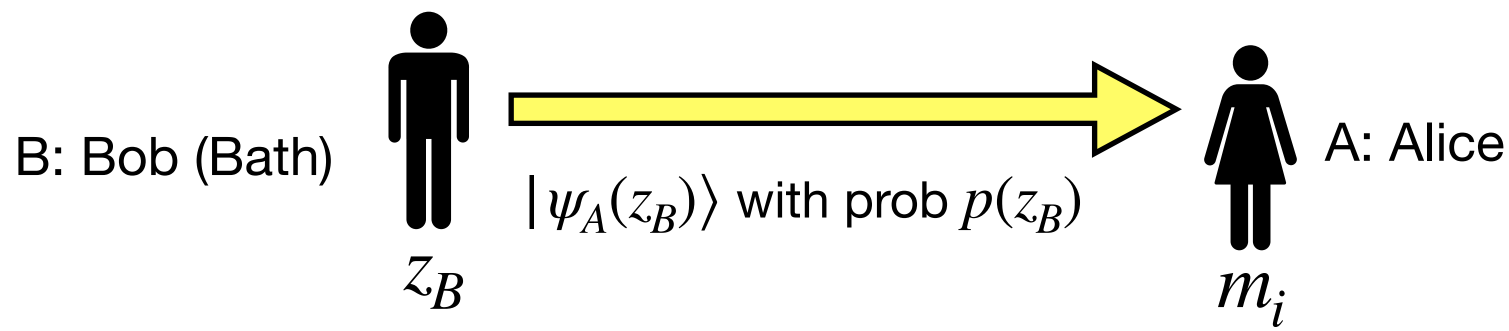

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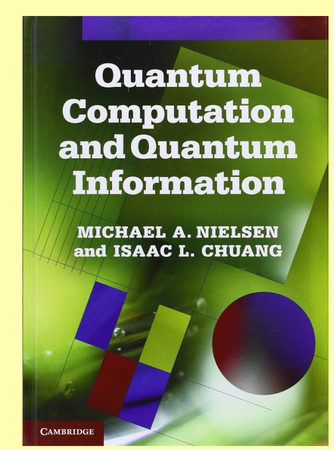
$$I_{acc}(\mathscr{E}) := \sup_{M \in POVM} I(\mathscr{E} : M)$$

"Maximal classical information extractable from quantum ensemble"



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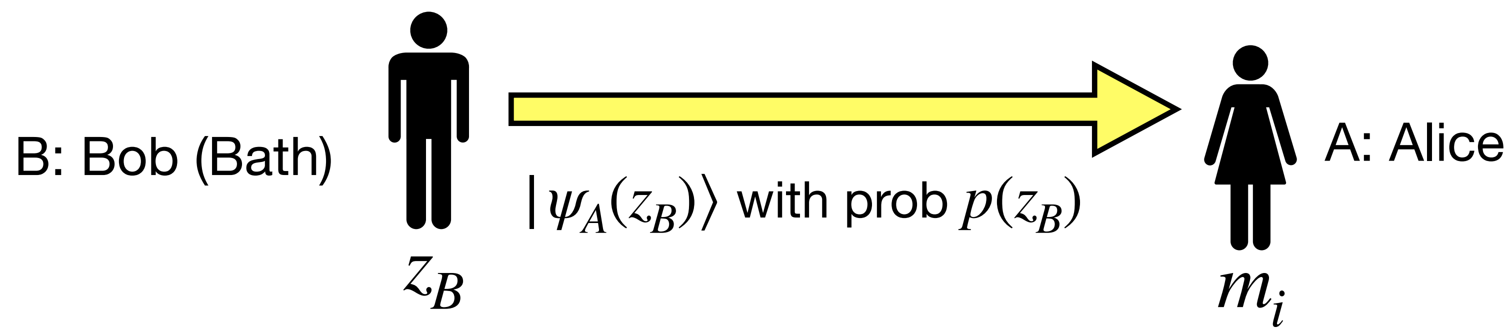

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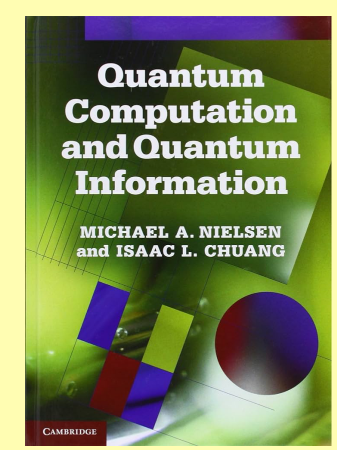

Q: What should we expect the accessible information of the projected ensemble to be?

Bounds on accessible information

Jozsa, Robbs, Wootters '94
bound: subentropy

$$Q(\rho) = -\sum_{k=1}^{n} \left(\prod_{l \neq k} \frac{\lambda_k}{\lambda_k - \lambda_l} \right) \lambda_k \ln \lambda_k$$

$$Q(\rho) \leq I_{acc}(\mathscr{E}) \leq S(\rho)$$

Holevo bound: Von
Neumann entropy

$$S(\rho) = -\sum_{k=1}^{n} \lambda_k \ln \lambda_k$$

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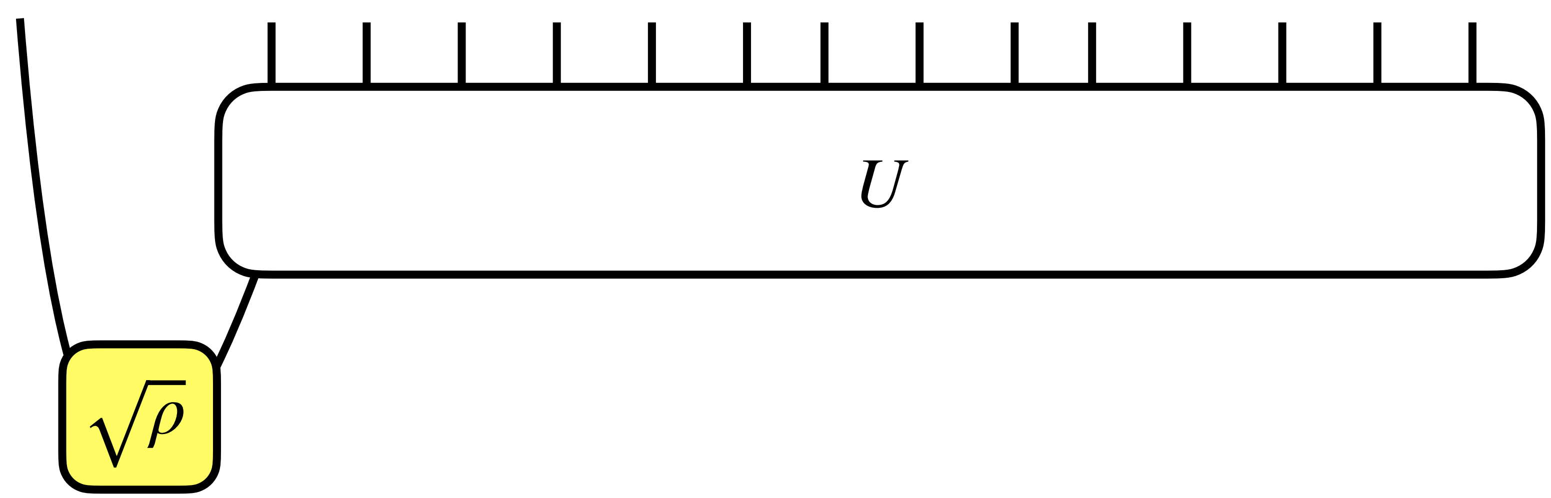

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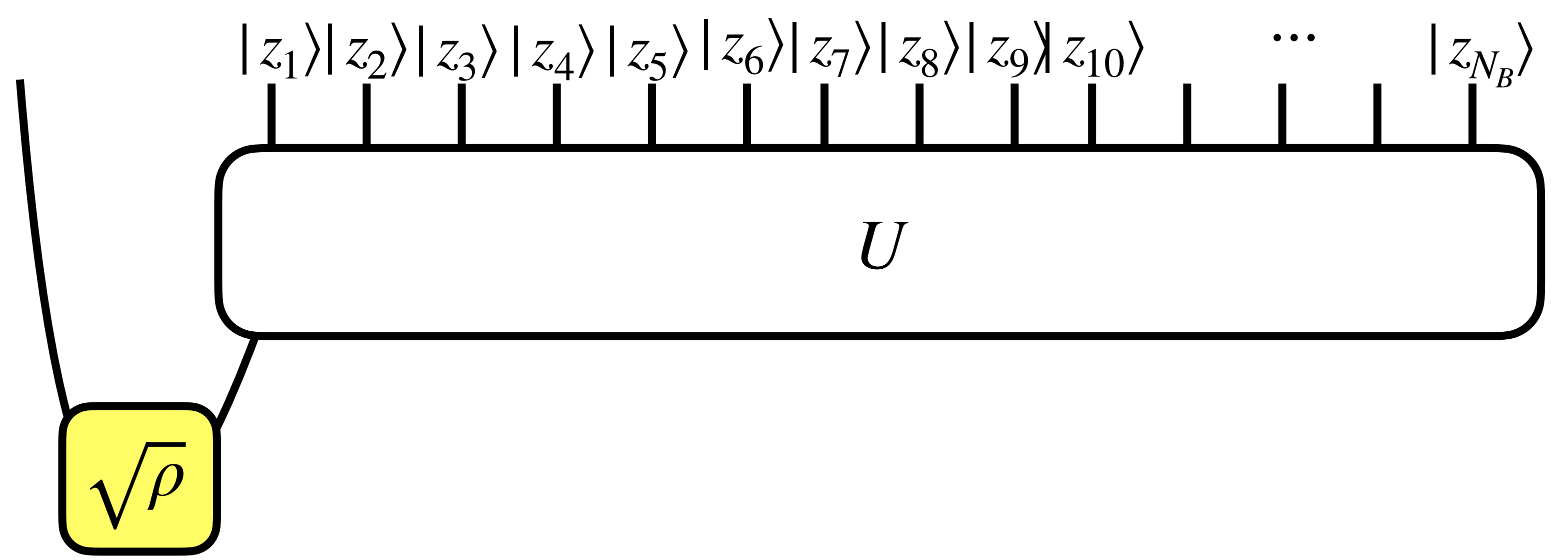


Bounds on accessible information

Jozsa, Robbs, Wootters '94 bound: subentropy

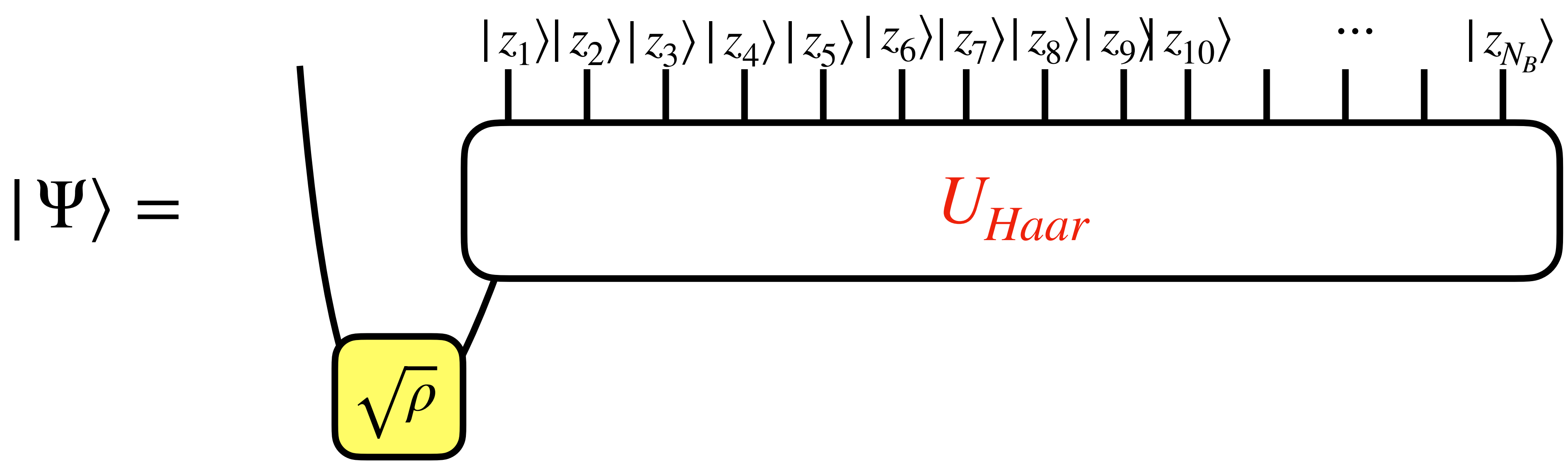
$$Q(\rho) = -\sum_{k=1}^{n}\left(\prod_{l\neq k}\frac{\lambda_k}{\lambda_k-\lambda_l}\right)\lambda_k \ln \lambda_k$$

$$Q(\rho) \leq I_{acc}(\mathscr{E}) \leq S(\rho)$$

Holevo bound: Von Neumann entropy

$$S(\rho) = -\sum_{k=1}^{n}\lambda_k \ln \lambda_k$$

- For projected ensemble, $\rho = \rho_A$= reduced density matrix on A, determined by *standard quantum thermalization*



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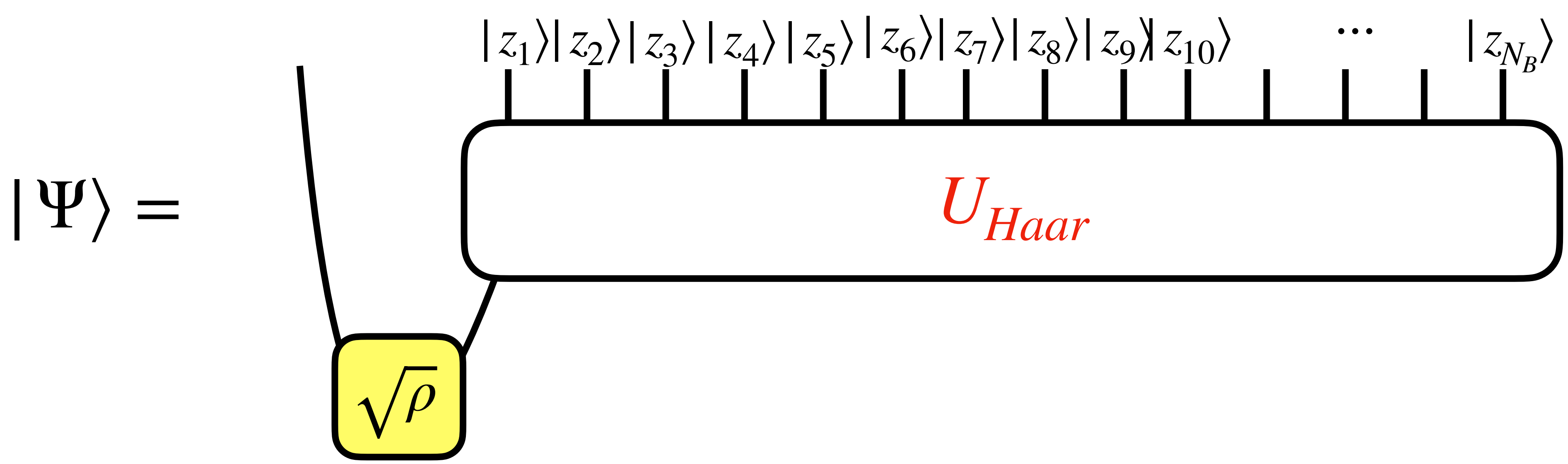

Our intuition: if nature is scrambling, we expect minimal information transmission!
I.e., nature is a lousy quantum communications channel =
"Deep thermalization"

Generalized maximum entropy principle for deep thermalization = Principle of minimal accessible information

[C Liu, QC Huang, WWH, arXiv: 2405.05470], Also [Mark et al, arXiv: 2403.11970]

$$\mathscr{E}^* = \text{arg max}_{\mathscr{E}} S(\mathscr{E}) \quad \text{s.t.} \quad \text{Mean}(\mathscr{E}) = \rho_A$$

$$S(\mathscr{E}) := - I_{acc}(\mathscr{E})$$

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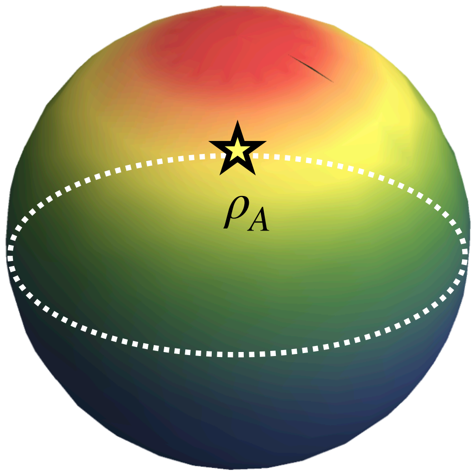
- For quantum spin systems, this limiting ensemble was derived by Jozsa, Robbs, Wootters [PRA '94], and is known as the **Scrooge ensemble**



Unraveling of ρ_A into an ensemble of pure states which yields the least information (i.e. most stingy)

Scrooge ensemble $\mathscr{E}_{Scr.}(\rho)$

1. Let $|\psi\rangle$ be a normalized state.
2. Distort state $|\psi\rangle \to \sqrt{\rho}\,|\psi\rangle$ ("rho-distortion") and define normalized state $|\Psi(\psi)\rangle := \sqrt{\rho}\,|\psi\rangle/\sqrt{\langle\psi|\rho|\psi\rangle}$
3. Sample $|\Psi(\psi)\rangle$ with probability density $p(\psi)d\psi = D\langle\psi|\rho|\psi\rangle d\psi$ where $d\psi$ is the uniform Haar measure and D the dimension of the Hilbert space

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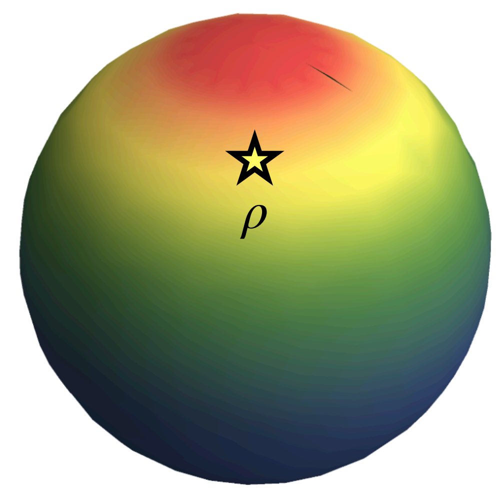
$$\mathscr{E}_{Scr.}(\rho) = \left\{ d\psi \underbrace{D\langle\psi|\rho|\psi\rangle}_{p(\psi)}, \underbrace{\frac{\sqrt{\rho}\,|\psi\rangle}{\langle\psi|\rho|\psi\rangle}}_{|\Psi(\psi)\rangle} \right\}$$



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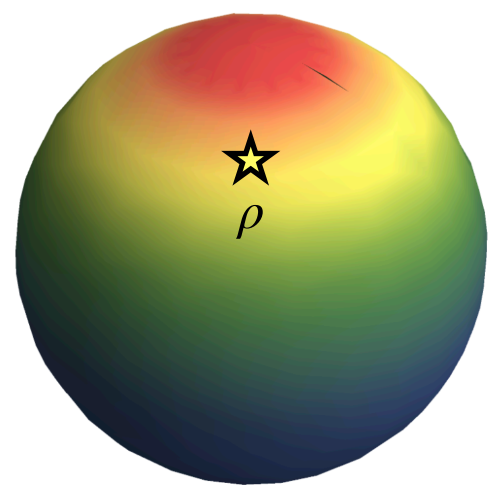
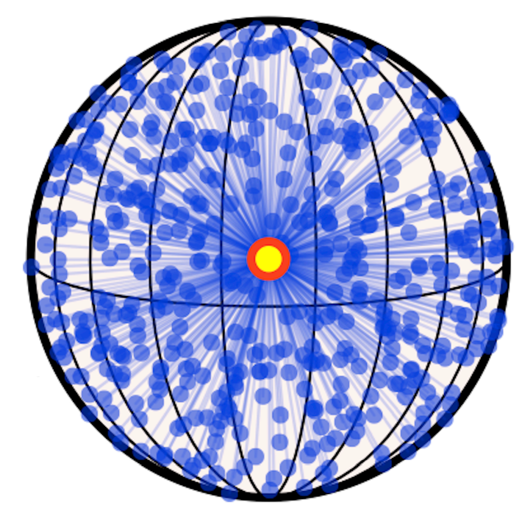
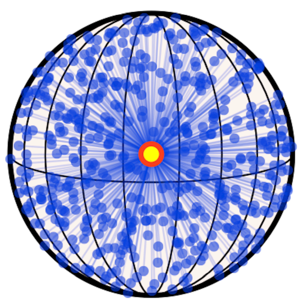




(Note: $\mathscr{E}_{Scr.}(\mathbb{I}/d) = \mathscr{E}_{Haar}$ i.e. uniform or Haar ensemble)

Emergence of universal ensembles in quantum many-body systems



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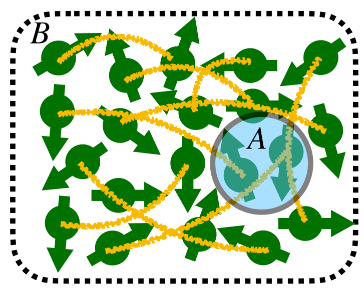
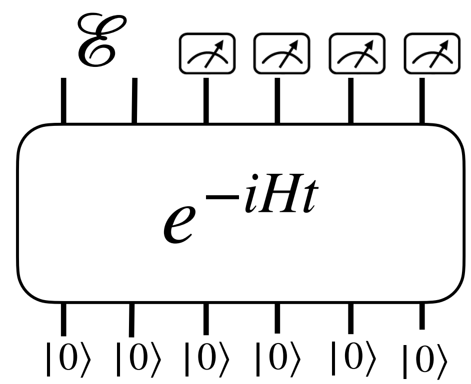
Haar random state
(Analytical proof)



[Cotler et al, PRXQ '23]

Quench dynamics at ∞-temperature
(Numerics, Experiments)



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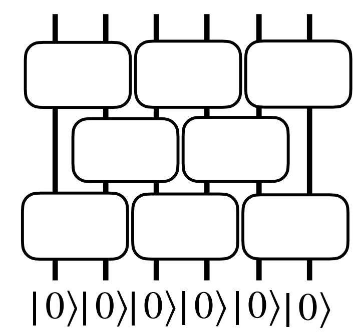

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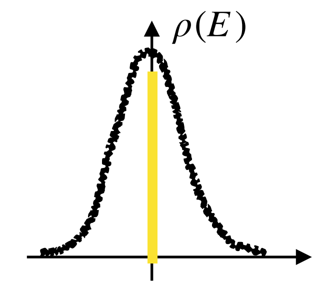

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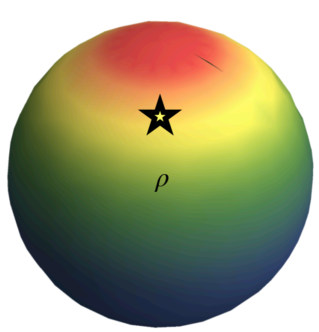

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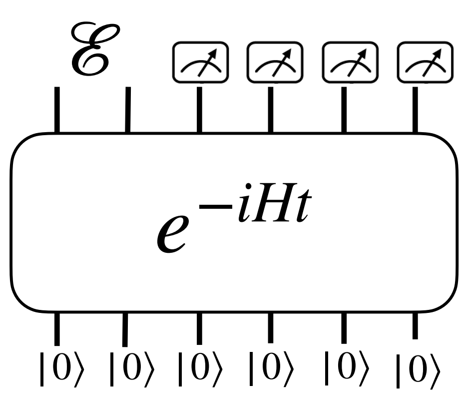

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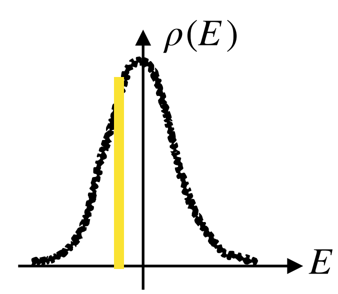

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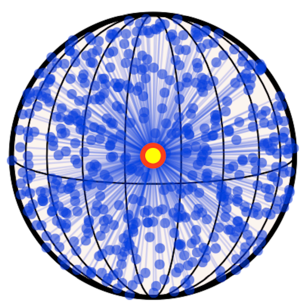

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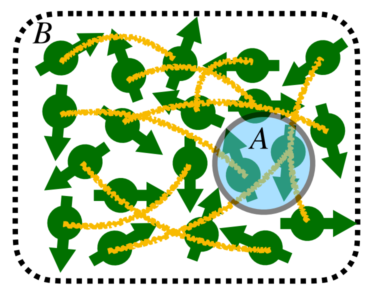

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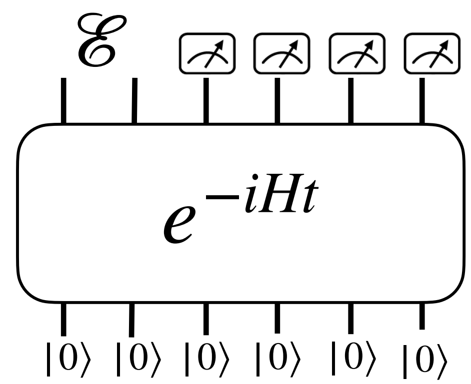

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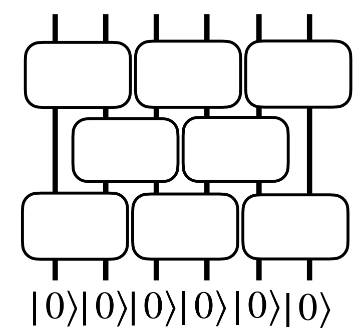

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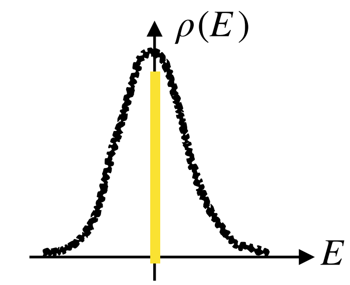

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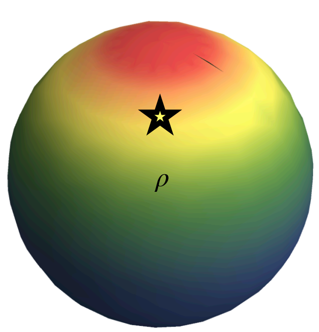

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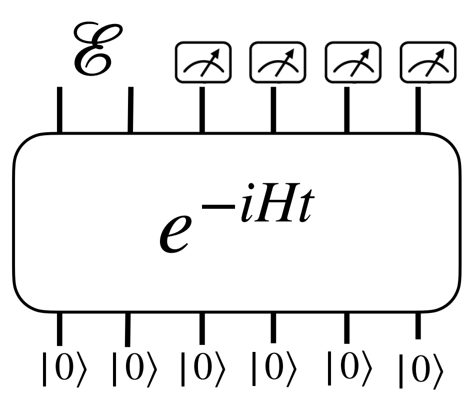

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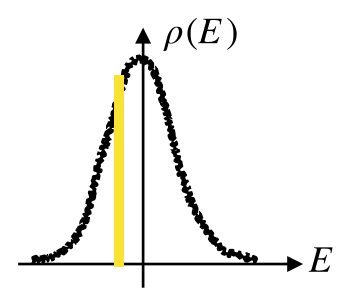

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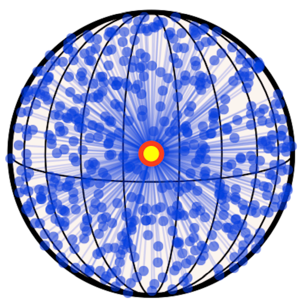

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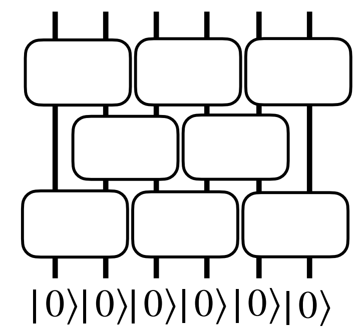

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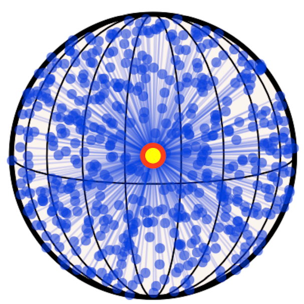

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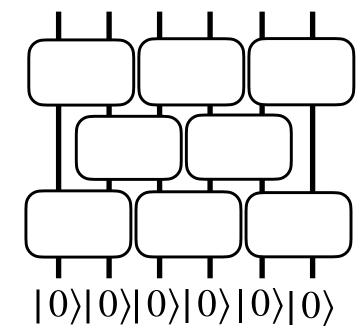

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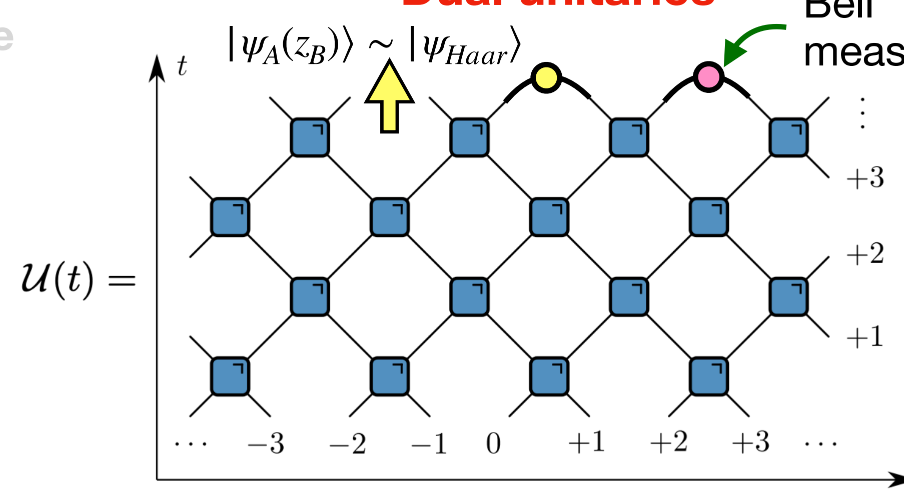
Dual unitaries



Key idea: Haar random states come from stochastic quantum computation in space, induced by measurements (akin to measurement-based quantum computation)

Previous studies have been confined to spin (and fermion) systems, with local bounded Hilbert space.

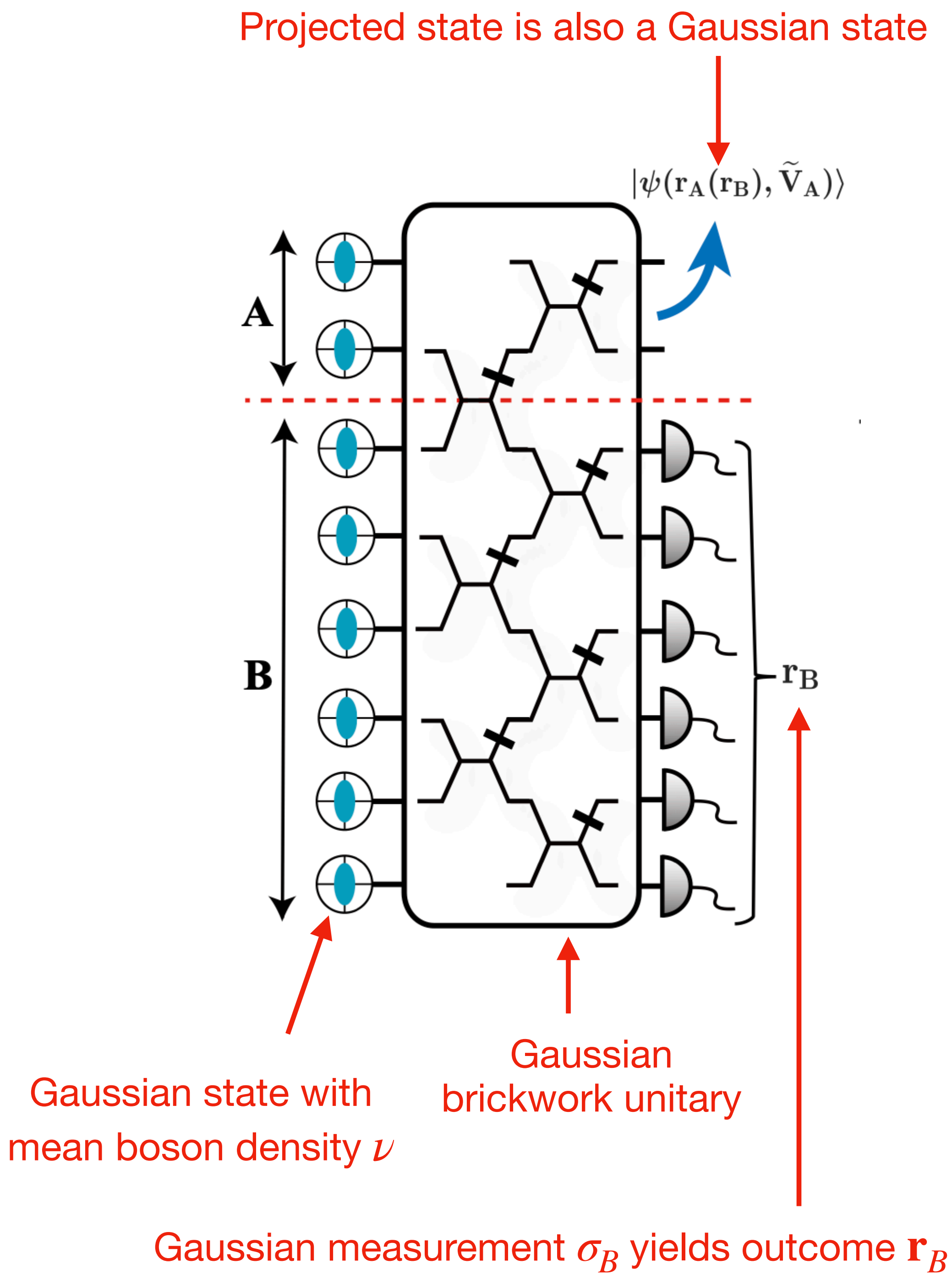
Q: Do the deep thermalization universality and generalized maximum entropy principle apply in a physically distinct system, e.g. systems of many bosons?

Continuous-variable system = Unbounded Hilbert space $\Longrightarrow$
no notion of Haar random states to 'distort' to construct Scrooge ensemble

(For the experts: no "2-designs" [Iosue, Sharma, Gullans, Albert, PRXQ 14,011013 (2024)])

[Liu, Huang, WWH, arXiv: 2405.05470]

Deep thermalization in Gaussian Continuous-Variable (CV) systems



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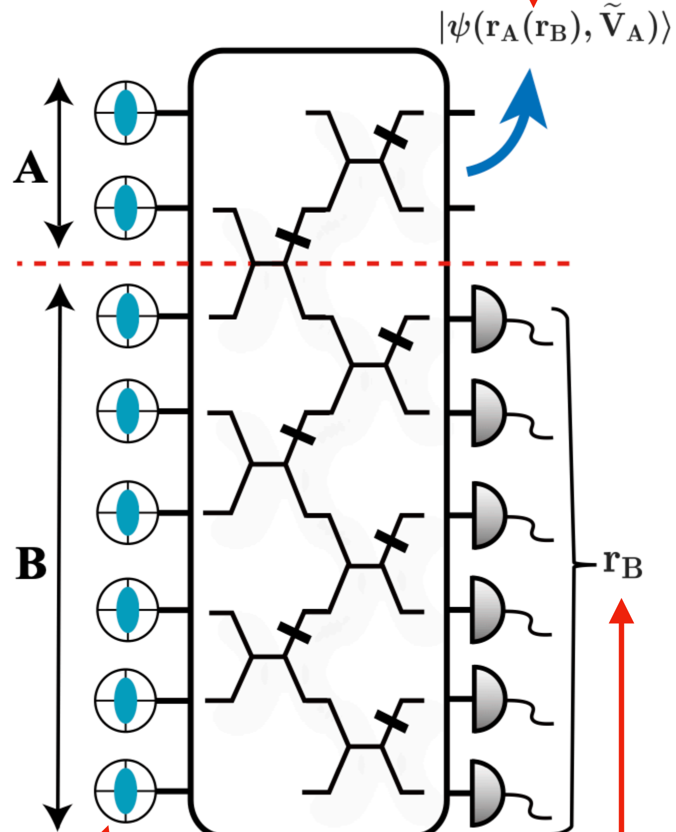
Deep thermalization in Gaussian Continuous-Variable (CV) systems

Projected state is also a Gaussian state



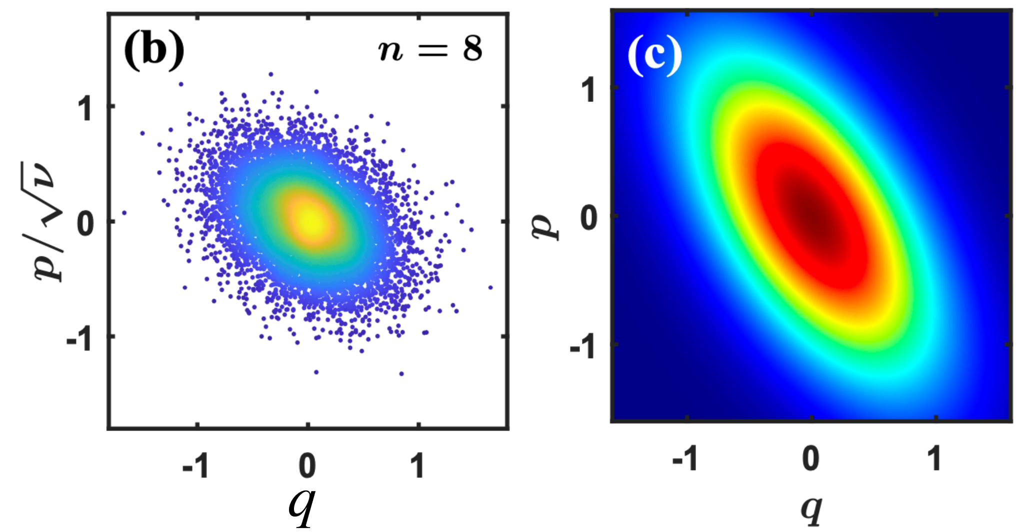
Gaussian state with mean boson density ν

Gaussian brickwork unitary

Gaussian measurement σ_B yields outcome $\mathbf{r}_B$

Distribution of $\mathbf{r}_A$

Wigner function of centered projected state



$$\mathcal{E}_G = \{p(\mathbf{r}_A), |\psi(\mathbf{r}_A, \tilde{V}_A)\rangle\}$$

Projected ensemble is a distribution of Wigner functions, characterised by displacement $\mathbf{r}_A$ and quantum covariance matrix $\tilde{V}_A$

[Liu, Huang, WWH, arXiv: 2405.05470]

Deep thermalization in Gaussian Continuous-Variable (CV) systems

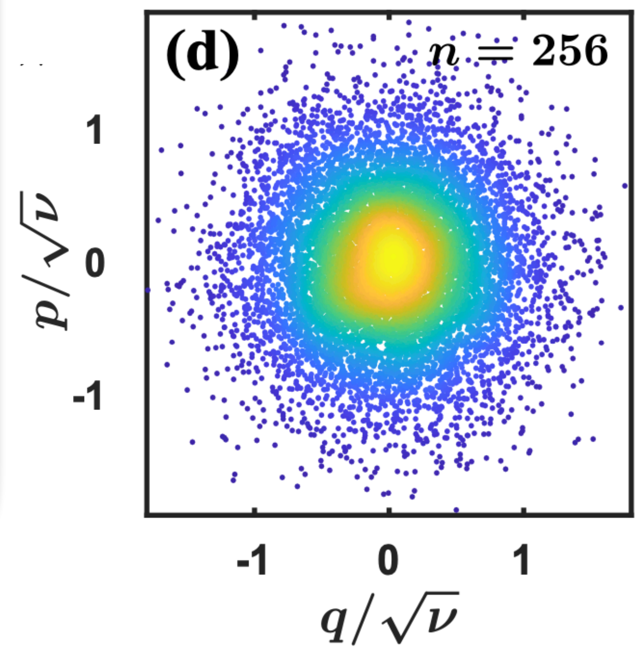
Our claim: universality arises in thermodynamic limit

Distribution of displacements: $\mathbf{r}_A \xrightarrow{d} \mathcal{N}(0,\, \nu \mathbb{I}_A)$

Covariance of each projected state: $\widetilde{V}_A \to \mathbb{I}_A$

Collection of unsqueezed coherent states

Distribution of $\mathbf{r}_A$



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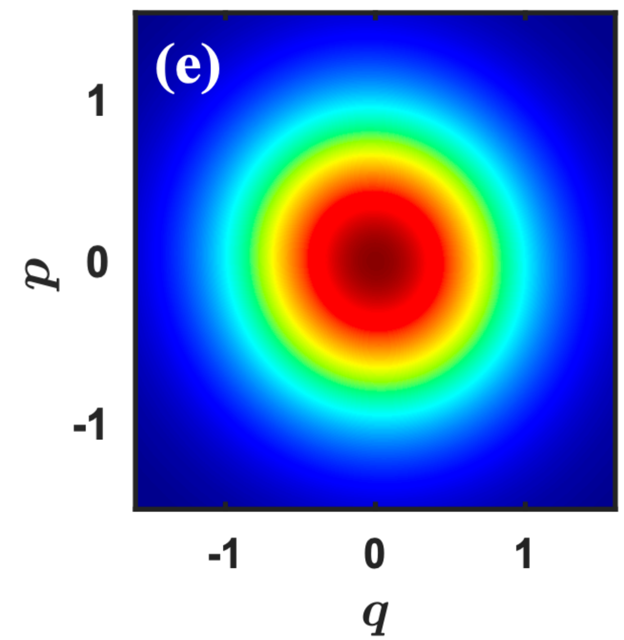

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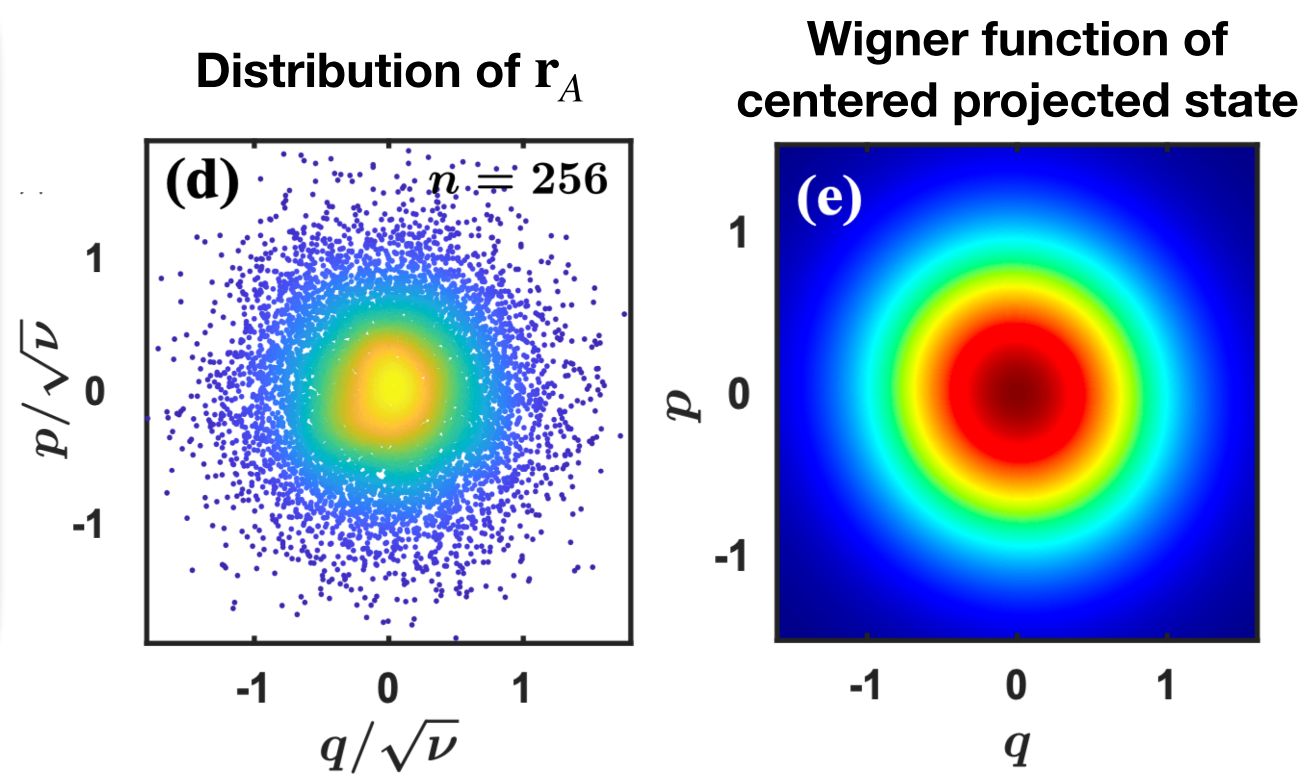

- ✓ Rigorous proof for random Gaussian states with mean ν
- ✓ Numerical evidence in brickwork circuit models
- ✓ Limiting ensemble has **minimal accessible information** which we call "*Gaussian Scrooge Distribution*" (analog of Scrooge in spin systems but for Gaussian CV systems) c.f. [Holevo, J. Math. Phys. 62, 092201 (2021)]

[Chang, Shrotriya, WWH, Ippoliti, 2408.15325]

Symmetries and deep thermalization

Q: How do symmetries change the universal limiting distribution?

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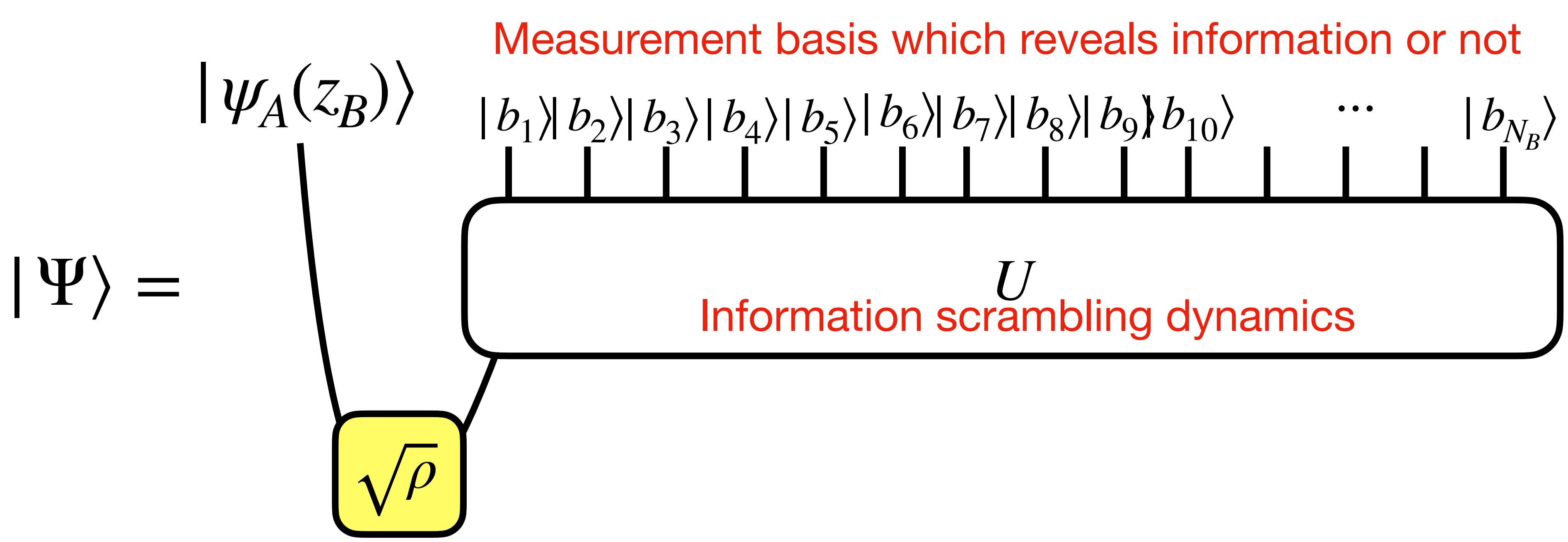

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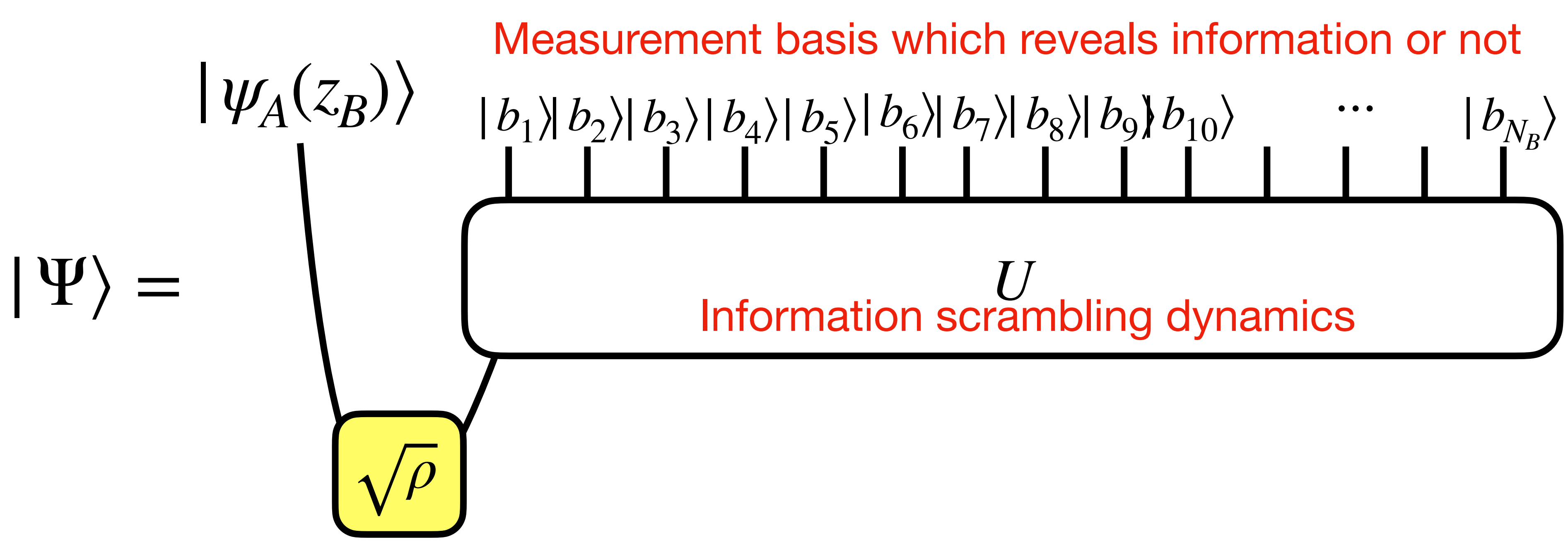

- Also, have to account for charge fluctuations in initial state!

[Chang, Shrotriya, WWH, Ippoliti, 2408.15325]
Also [Mark et al, arXiv: 2403.11970]

Measurement basis matters!

To be concrete, consider a local on-site symmetry $[Q, U] = 0$, e.g. $Q = \sum_i Z_i$.

Then a global state can be written $|\Psi\rangle = \sum_Q p(Q)|\Phi_Q\rangle$

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Charge non-revealing, e.g. $\{ |x\rangle \}$

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(Normal) Scrooge ensemble

- Assuming $p(Q) = \delta_{Q,Q_0}$
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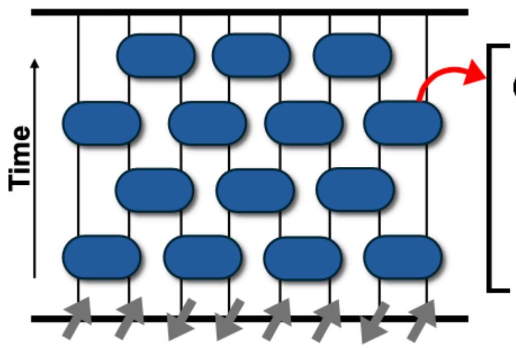
Charge revealing $\{ |z\rangle \}$

$$\mathscr{E}_{PE} \to \sum_{Q_B} \pi(Q_B)\mathscr{E}_{Scr.}(\rho_A(Q_B))$$

Generalized Scrooge ensemble

- Convex sum of Scrooges, depending on charge measured Q_B
- Represents stingy unraveling up to Bayesian update of belief of state on A
- Universal: only depending on charge distribution $p(Q)$

Numerical confirmation



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Random, charge-conserving U(1) circuits

Initial product state: $|\Psi(0)\rangle = [(\cos\theta|0\rangle + \sin\theta|1\rangle) \otimes (\sin\theta|0\rangle + \cos\theta|1\rangle)]^{\otimes \frac{N}{2}}$

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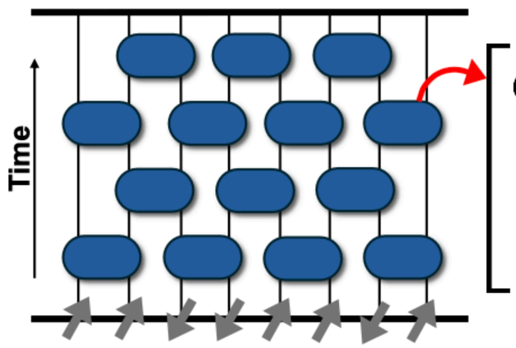

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Compute trace distance of k = 2 moment between ensembles

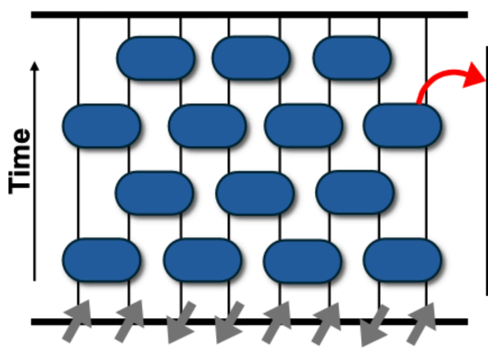
$$\rho_{\mathscr{E}}^{(k)} = \mathbb{E}_{\phi\sim\mathscr{E}}(\,|\,\phi\rangle\langle\phi\,|\,)^{\otimes k}$$

All k-point correlations of an ensemble

$$\Delta^{(k)}(\mathscr{E}, \mathscr{E}') := \frac{1}{2}\,\left\|\,\rho_{\mathscr{E}}^{(k)} - \rho_{\mathscr{E}'}^{(k)}\,\right\|_1$$

Trace distance = Optimal distinguishability
with k-copies of state between ensembles

Numerical confirmation



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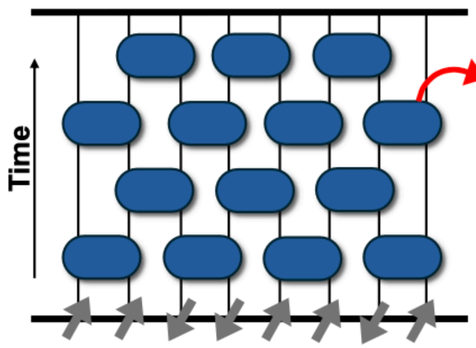

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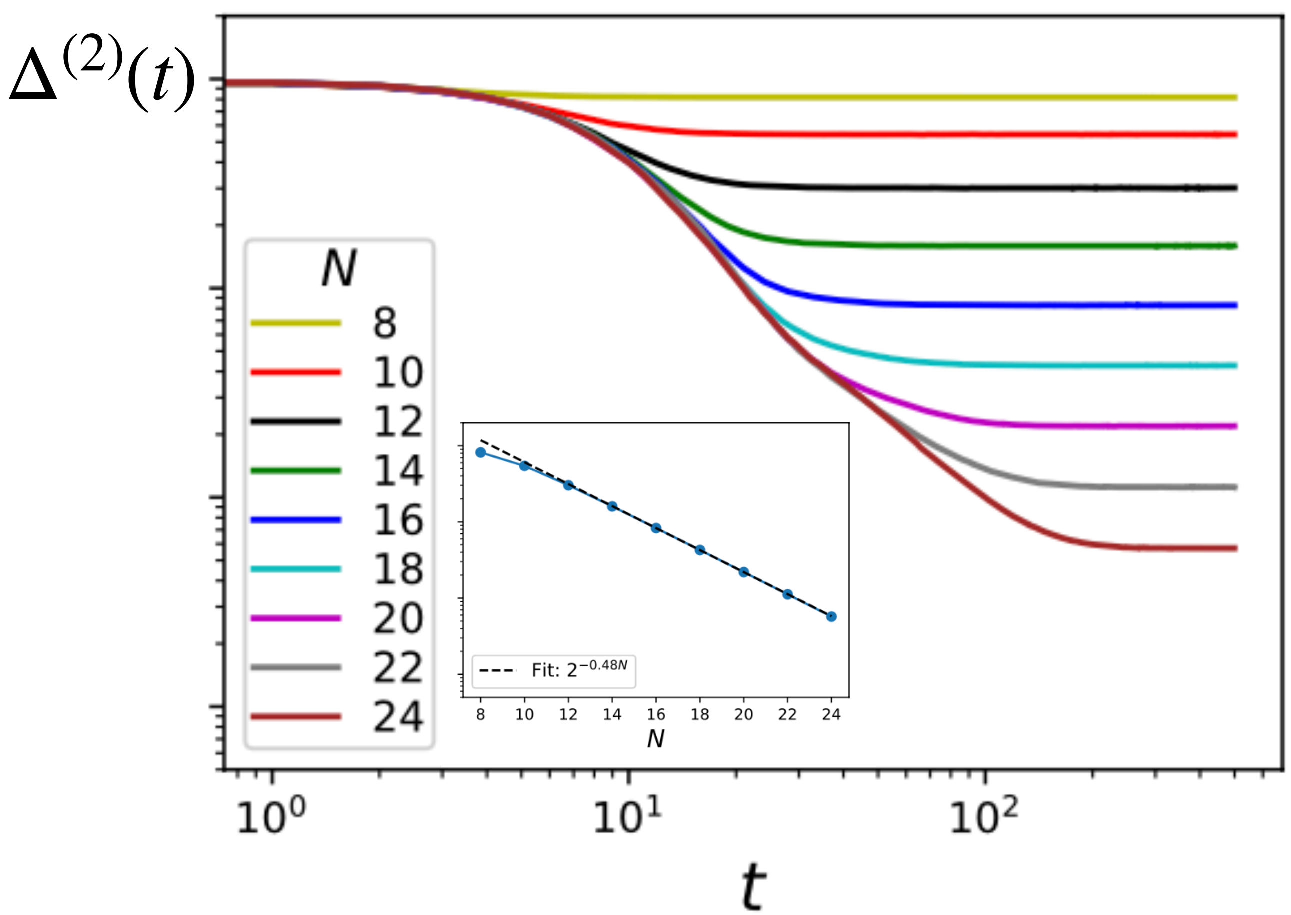
Compute trace distance of k = 2 moment between ensembles

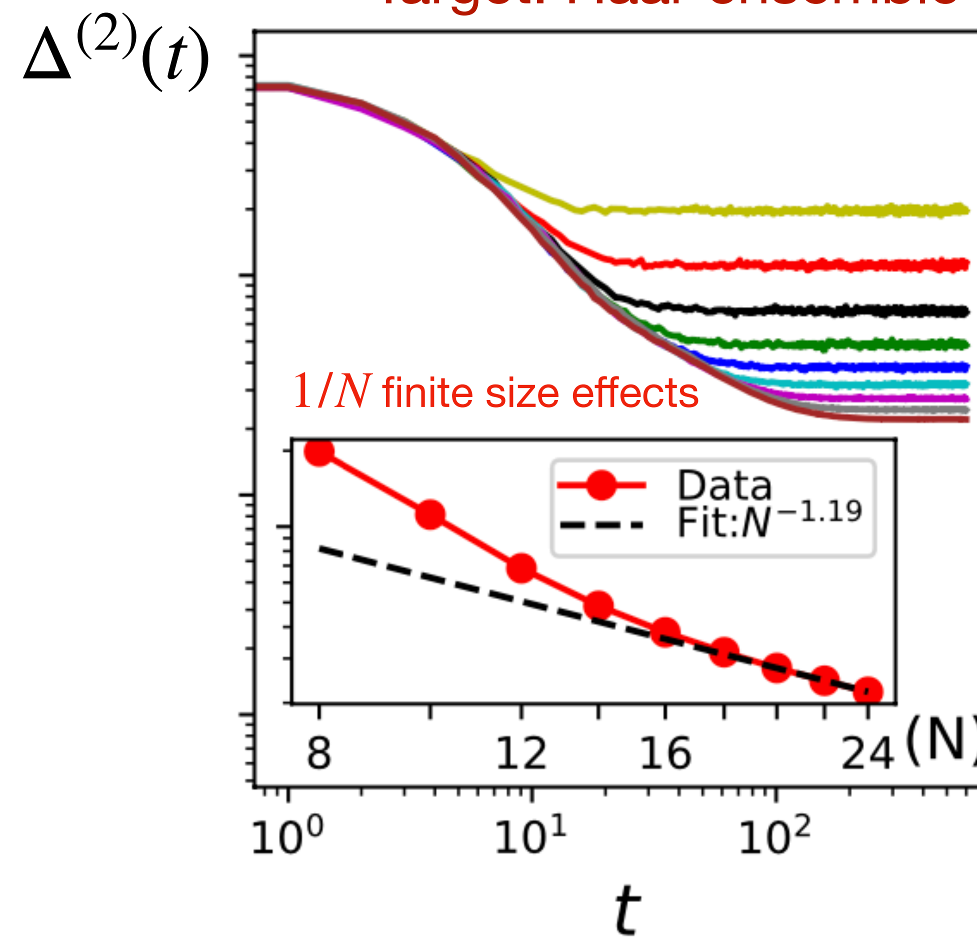
$\theta = \pi/20$; z-basis measurement

Target: Generalized Scrooge ensemble



$\theta = 0$ (Neel); x-basis measurement

Target: Haar ensemble



Summary of current understanding of deep thermalization

Spins/Fermions

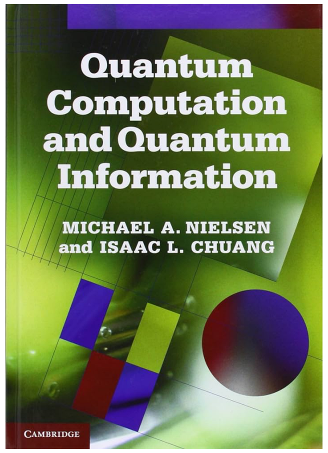
Constraints/ Conservation law	Charge distribution in initial state	Measurements in charge revealing basis	Measurements in charge non-revealing basis
None e.g. Floquet, circuits	N/A	N/A	$\mathscr{E}_{Haar}$
U(1) charge	None; $Q = N/2$ None; $Q \neq N/2$ Equilibrium; $\beta\mu = 0$ Equilibrium; $\beta\mu \neq 0$ General	$\oplus_{Q_A} p(Q_A)\mathscr{E}_{Haar}(Q_A)$ $\oplus_{Q_A} p(Q_A)\mathscr{E}_{Haar}(Q_A)$ $\mathscr{E}_{Haar}$ $\mathscr{E}_{Scr.}$ $\mathscr{E}_{Gen.\ Scr.}$ depending on $p(Q)$	$\mathscr{E}_{Haar}$ $\mathscr{E}_{Scr.}$ $\mathscr{E}_{Haar}$ (Numerical) $?$ $?$
Energy	Product state (Gaussian) at $\beta = 0$ Product state (Gaussian) at $\beta \neq 0$	$\mathscr{E}_{Gen.\ Scr.} \approx \oplus_{E_A} p(E_A)\mathscr{E}_{Haar}(E_A)$ $\mathscr{E}_{Generalized.\ Scr.}$	$\mathscr{E}_{Haar}$ $\mathscr{E}_{Scr.}$

Bosonic CV

		Measurements in any Gaussian basis	Measurements in Fock basis
Gaussian, U(1)	Squeezed product state with mean number ν	$\mathscr{E}_{Bos.\ Gaus.\ Scr.}$	$?$
Non-Gaussian, General	$?$	$?$	$?$

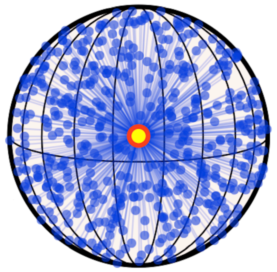
Conclusions

- Novel equilibration universality (Deep Thermalization) beyond standard quantum thermalization
- Underpinned by Generalized Maximum Entropy Principles from quantum information theory
- Rich host of emergent universal limiting ensembles constrained by symmetries



Take-home message:

New paradigms in quantum many-body dynamics from quantum information theory!



- **Questions:** Time-scales of deep thermalization; how is this related to entanglement and magic generation? Connections to OTOCs, quantum error correction? Applications for quantum information science?

Thank you for your attention!

Conclusions

arXiv > quant-ph > arXiv:2402.10181

Search...

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Quantum Physics

[Submitted on 15 Feb 2024]

Extracting randomness from quantum 'magic'

Christopher Vairogs, Bin Yan

Magic is a critical property of quantum states that plays a pivotal role in fault-tolerant quantum computation. Simultaneously, random states have emerged as a key element in various randomized techniques within contemporary quantum science. In this study, we establish a direct connection between these two notions. More specifically, our research demonstrates that when a subsystem of a quantum state is measured, the resultant projected ensemble of the unmeasured subsystem can exhibit a high degree of randomness that is enhanced by the inherent 'magic' of the underlying state. We demonstrate this relationship rigorously for quantum state 2-designs, and present compelling numerical evidence to support its validity for higher-order quantum designs. Our findings suggest an efficient approach for leveraging magic as a resource to generate random quantum states.

- **Questions:** Time-scales of deep thermalization; how is this related to entanglement and magic generation? Connections to OTOCs, quantum error correction? Applications for quantum information science?

Thank you for your attention!

Conclusions

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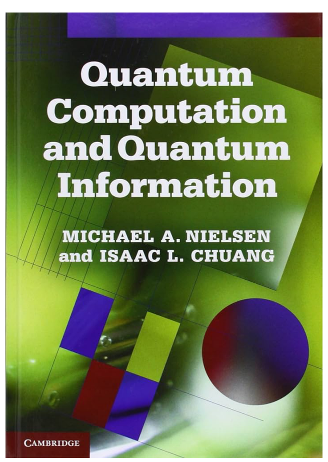

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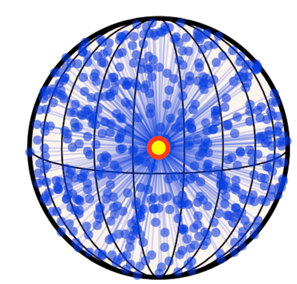

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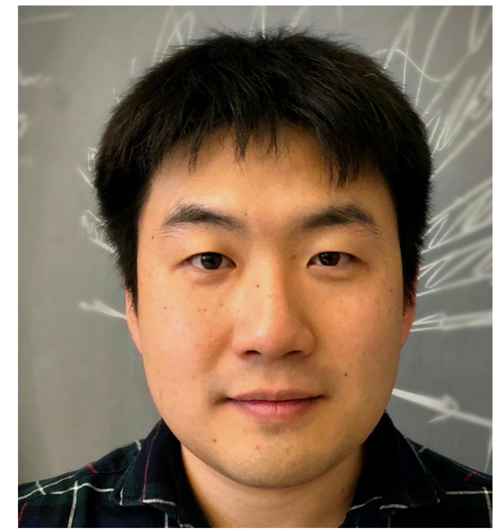
Thank you for your attention!

Acknowledgments

Harshank Shrotriya
CQT, NUS

Chang Liu
NUS

Qi Camm Huang
NUS



Soonwon Choi
MIT

Matteo Ippoliti
UT Austin

Thank you for your attention!

[WWH, S. Choi, PRL 128, 060621 (2022)]
[Ippoliti and WWH, PRX Q 4, 030322 (2023)]
[Ippoliti and WWH, Quantum 6, 886 (2022)]
[Shrotriya and WWH, arXiv:2305.08437]
[C Liu, QC Huang, WWH, arXiv: 2405.05470]

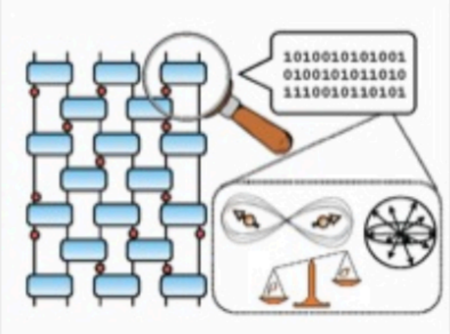
Learning the Fine Structure of Quantum Dynamics in Programmable Quantum Matter

Coordinators: Wen Wei Ho, Matteo Ippoliti, Joaquin Rodriguez Nieva, and Romain Vasseur

Scientific Advisors: Ehud Altman, David Huse, Monika Schleier-Smith, and Peter Zoller

Rapid advances in analog quantum simulators and digital quantum computers have opened up novel ways to control and interrogate quantum many-body systems. Such capabilities allow for the exploration of previously inaccessible dynamical regimes–like dynamics in the presence of monitoring and feedback–as well as to furnish new tools to learn features of complex quantum states and processes. New questions arise as to the novel universal phenomena that can be found in these new dynamical regimes, and about the optimal strategies and fundamental limitations of extracting information from such quantum systems.

This program has three main goals: (i) to chart the landscape of quantum dynamics in programmable and interactive quantum matter, (ii) to identify fundamental and practical limits on learning from quantum experiments, and (iii) to apply these ideas toward long-standing foundational questions of statistical mechanics such as chaos, ergodicity, and thermalization. The program aims to make progress on these interdisciplinary research frontiers by bringing together the communities of quantum condensed matter physics, quantum information theory, statistical physics, and atomic-molecular-optical physics.



DATES

Sep 29, 2025 - Oct 30, 2025

INFORMATION

Apply

Application deadline is:
Nov 24, 2024.
Primary deadline above date.

Rolling admissions after until the program is filled.

FAMILY SUPPORT INFO

Family Support Info

[Chang, Shrotriya, WWH, Ippoliti, 2408.15325]

Charge conservation: case study I

- Assume dynamics conserves charge $[U, Q] = 0, \quad Q = \sum \sigma_i^z$

- Assume initial state is random but has definite charge Q_0

[Chang, Shrotriya, WWH, Ippoliti, 2408.15325]

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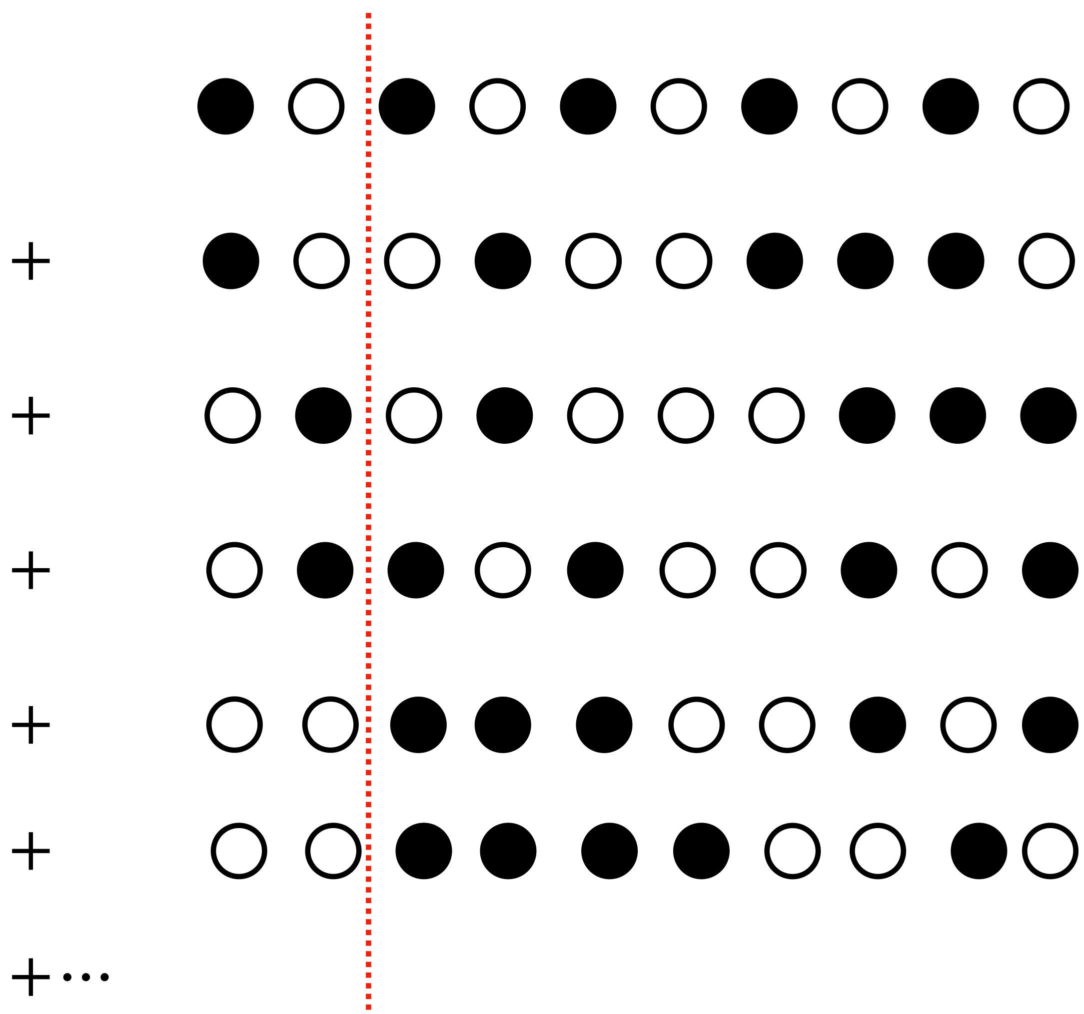

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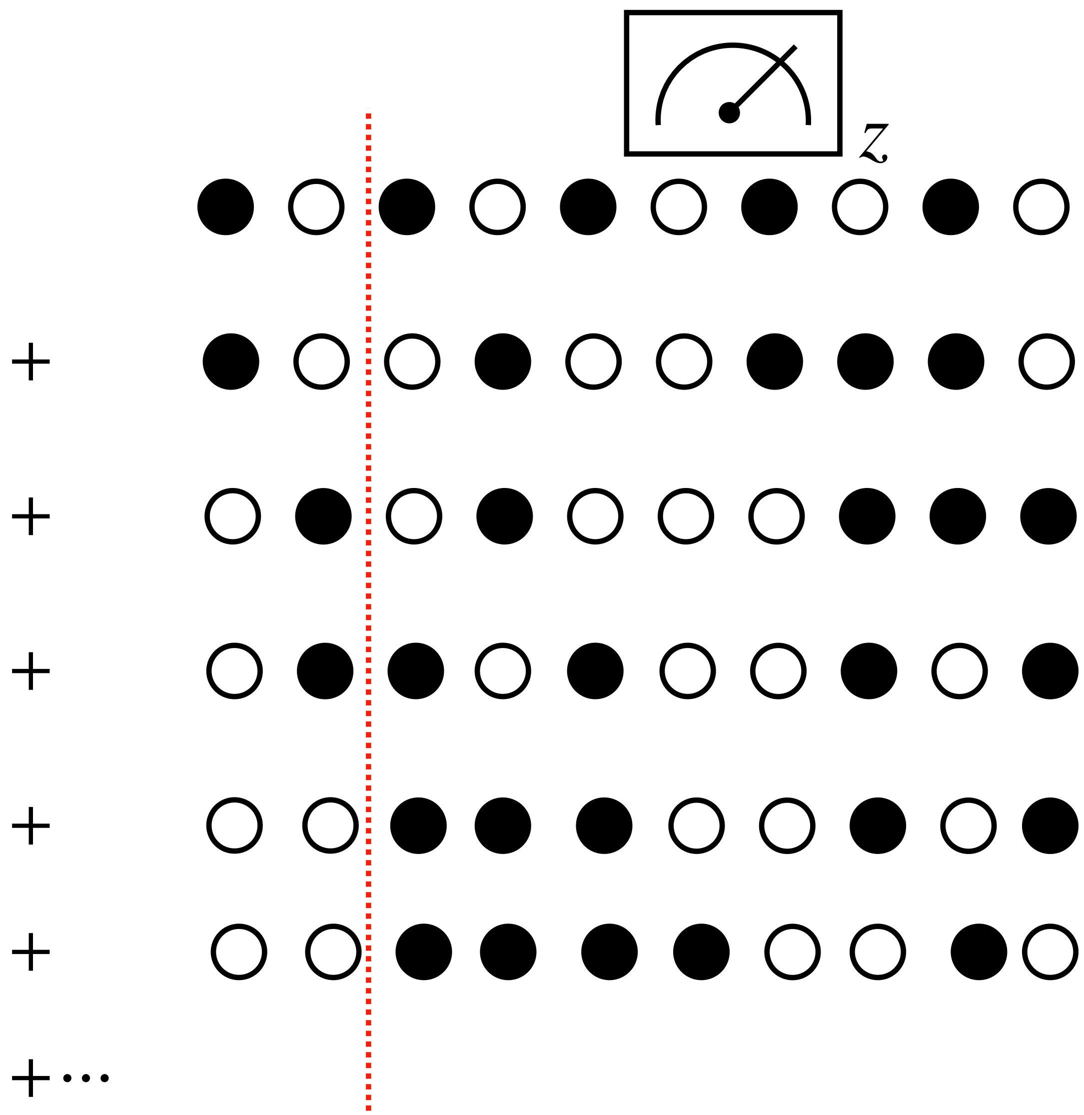

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[Chang, Shrotriya, WWH, Ippoliti, 2408.15325]

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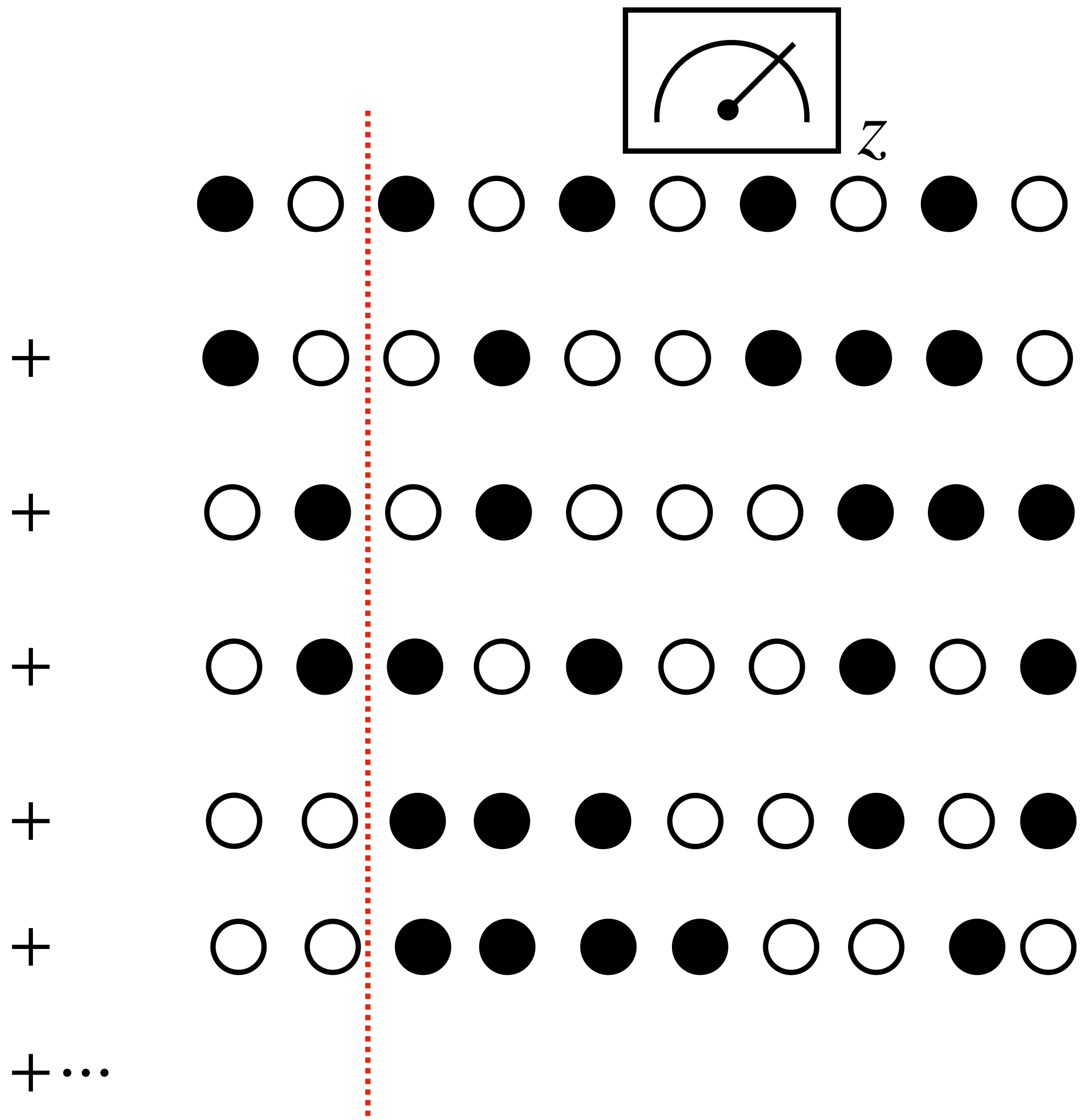

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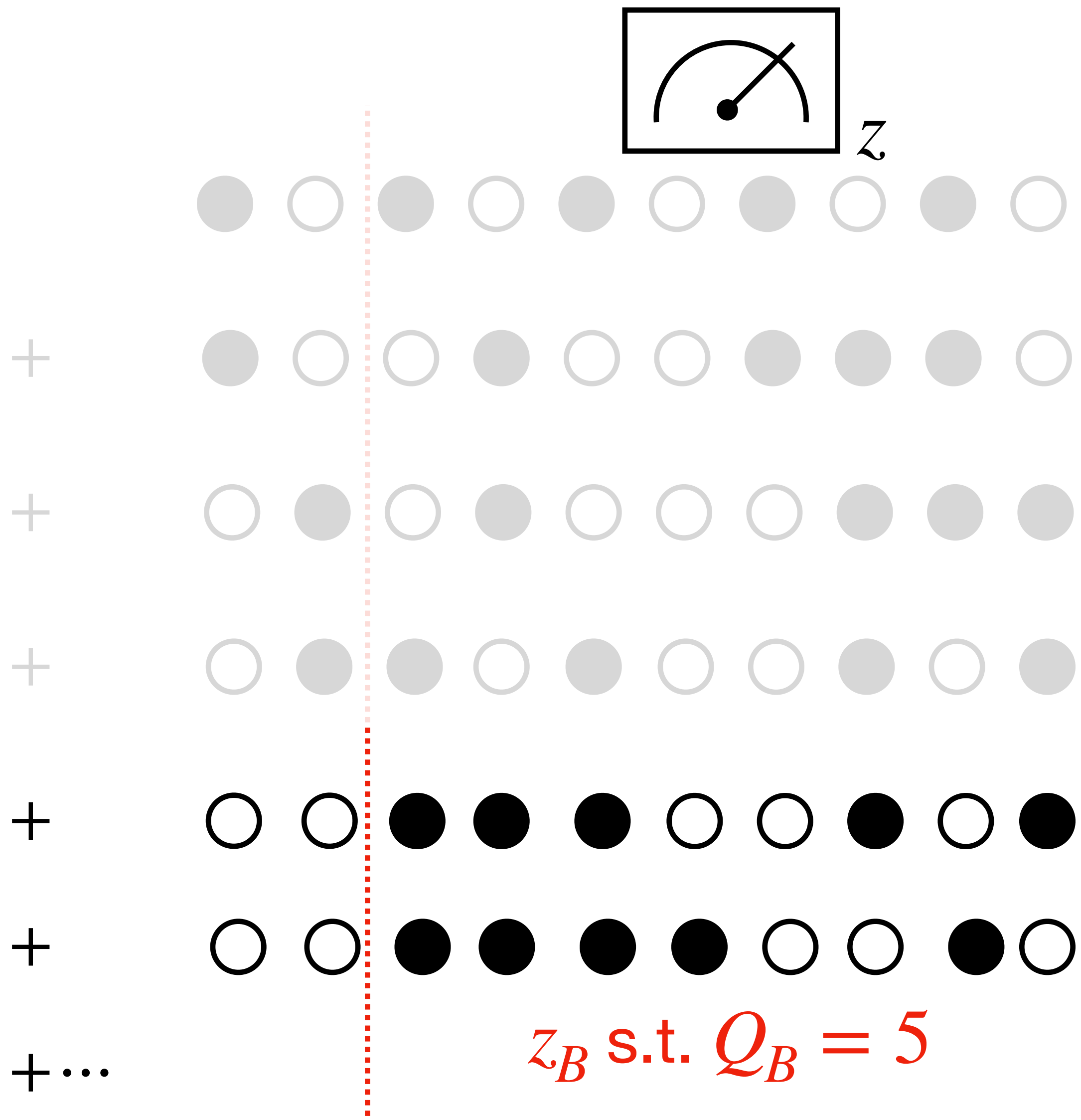

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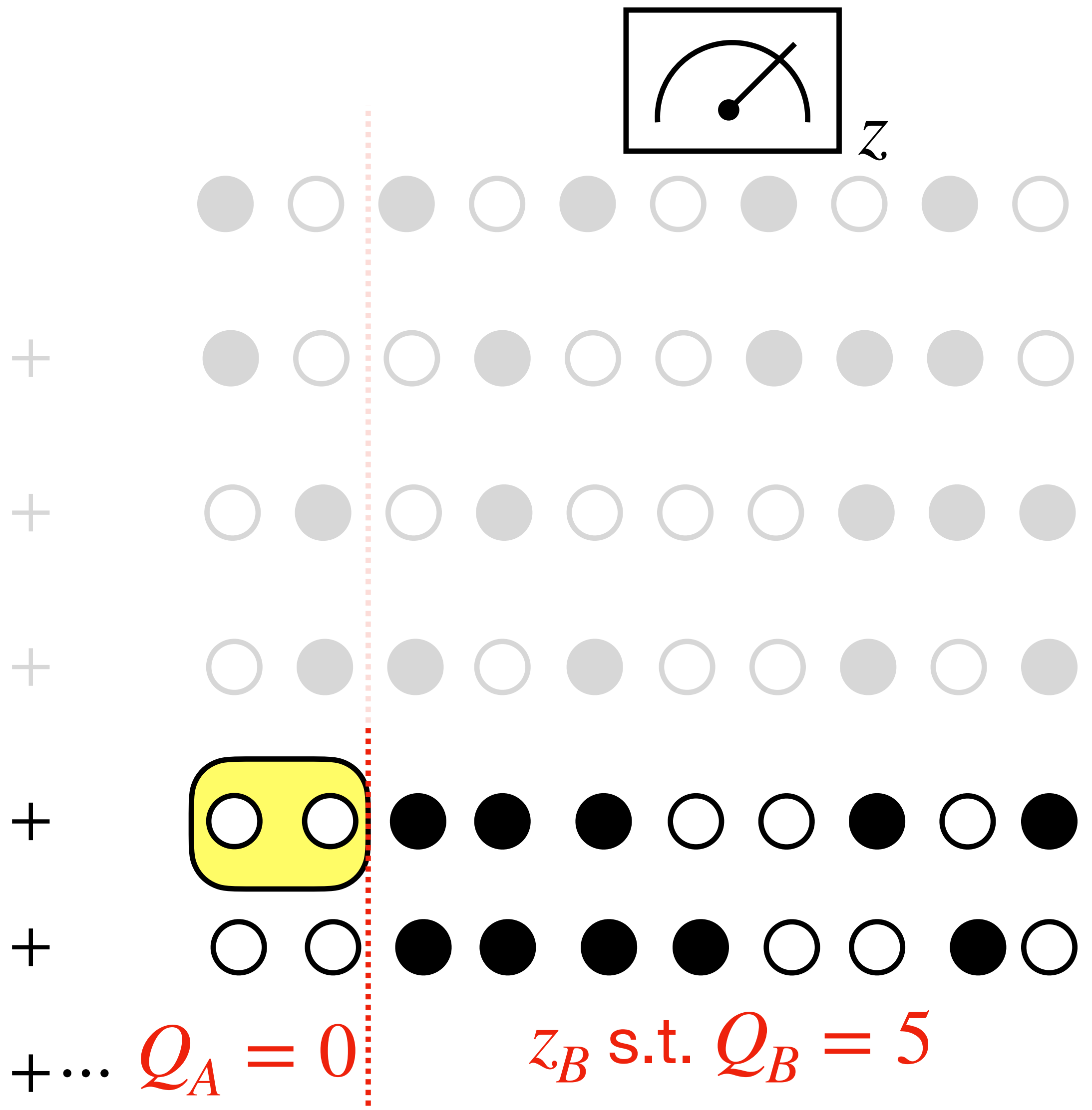

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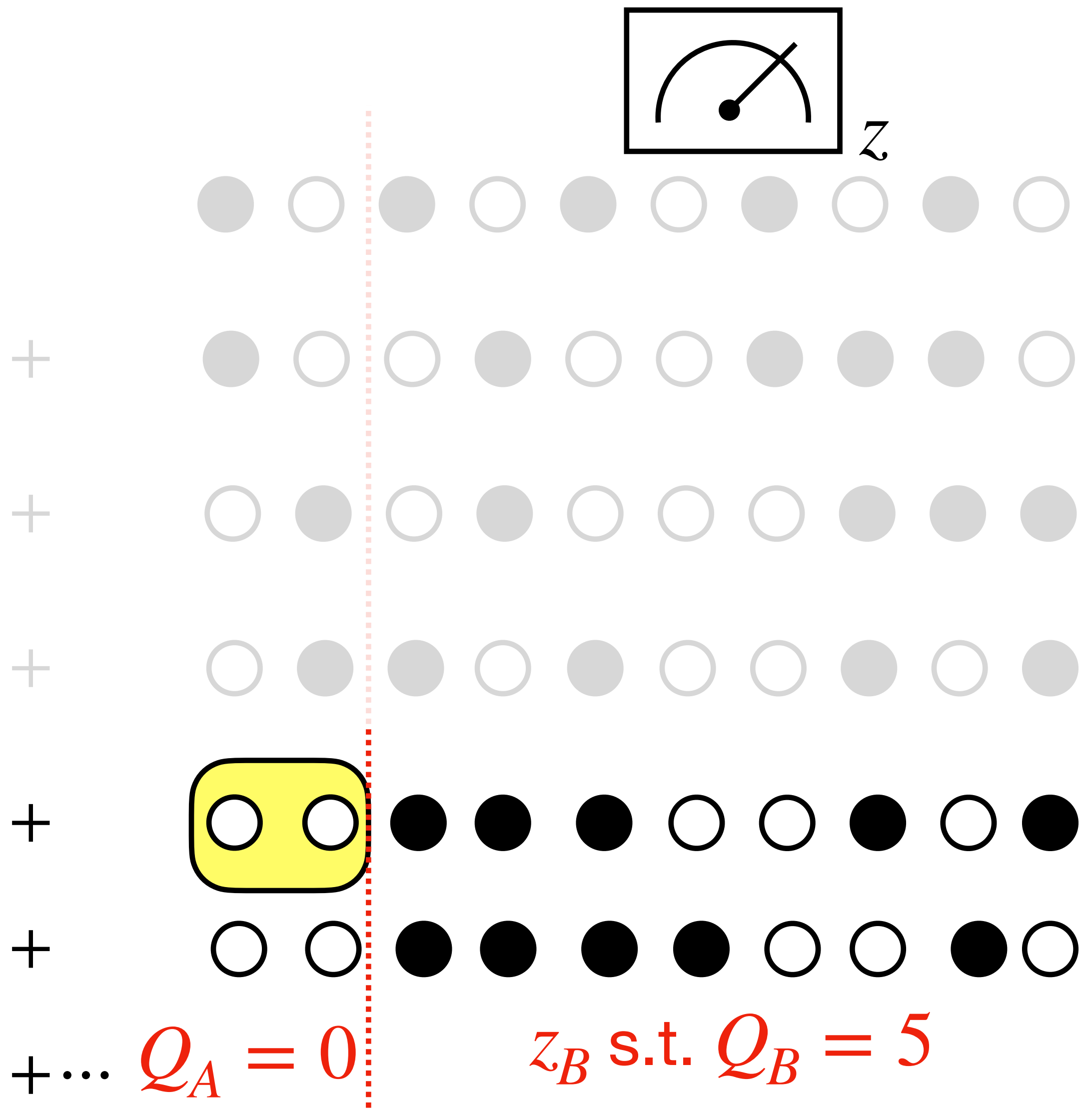

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- We thus expect limiting projected ensemble to block-diagonal (direct-sum):

$$\mathscr{E}_{PE} \to \bigoplus_{Q_A} p(Q_A) \mathscr{E}_{Haar,A}(Q_A)$$

We rigorously prove this statement in our paper

[Chang, Shrotriya, WWH, Ippoliti, 2408.15325]

Charge conservation: case study II

- Assume dynamics conserves charge $[U, Q] = 0, \quad Q = \sum \sigma_i^z$
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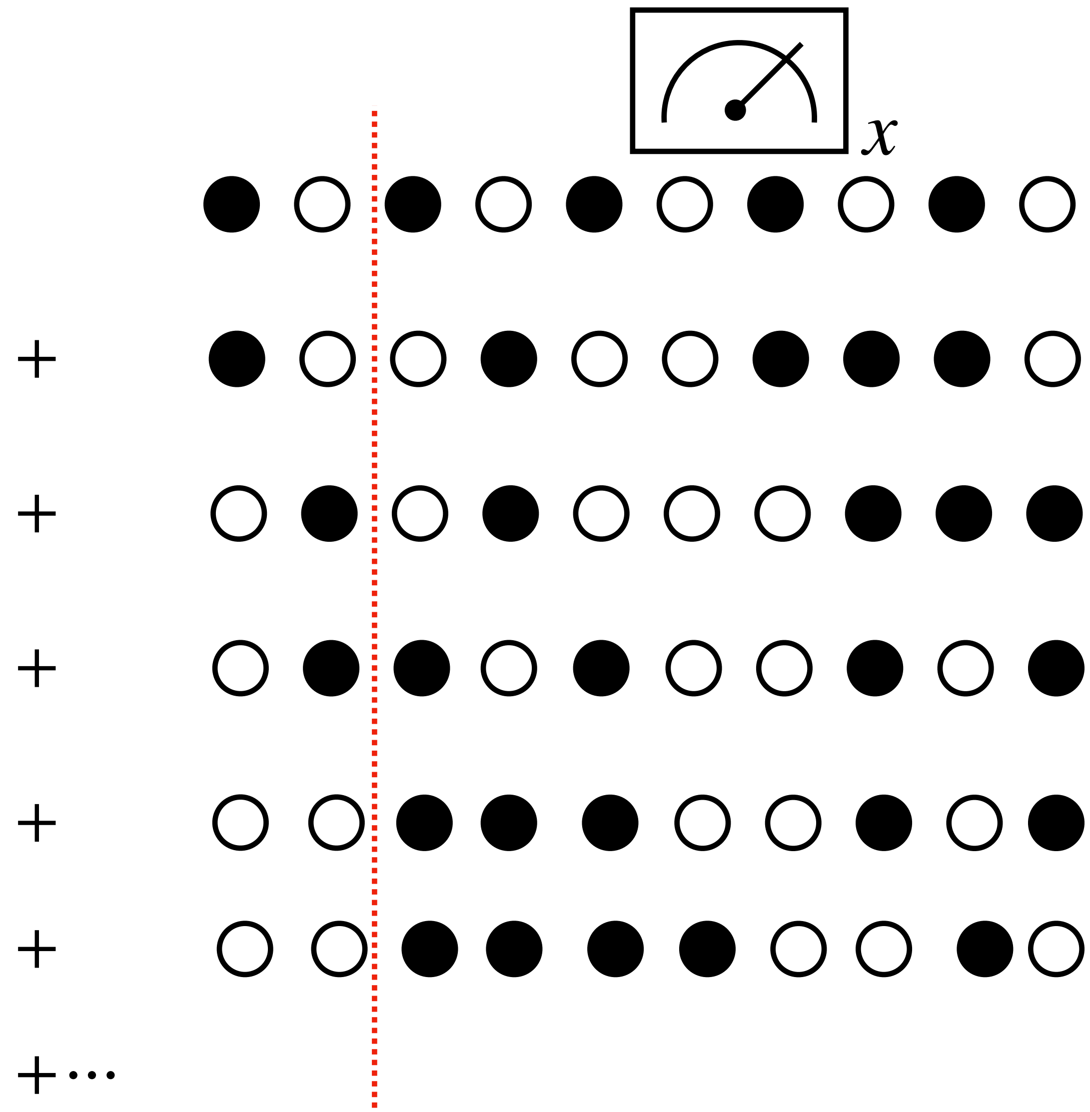

- Measure B in the computational basis x
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[Chang, Shrotriya, WWH, Ippoliti, 2408.15325]

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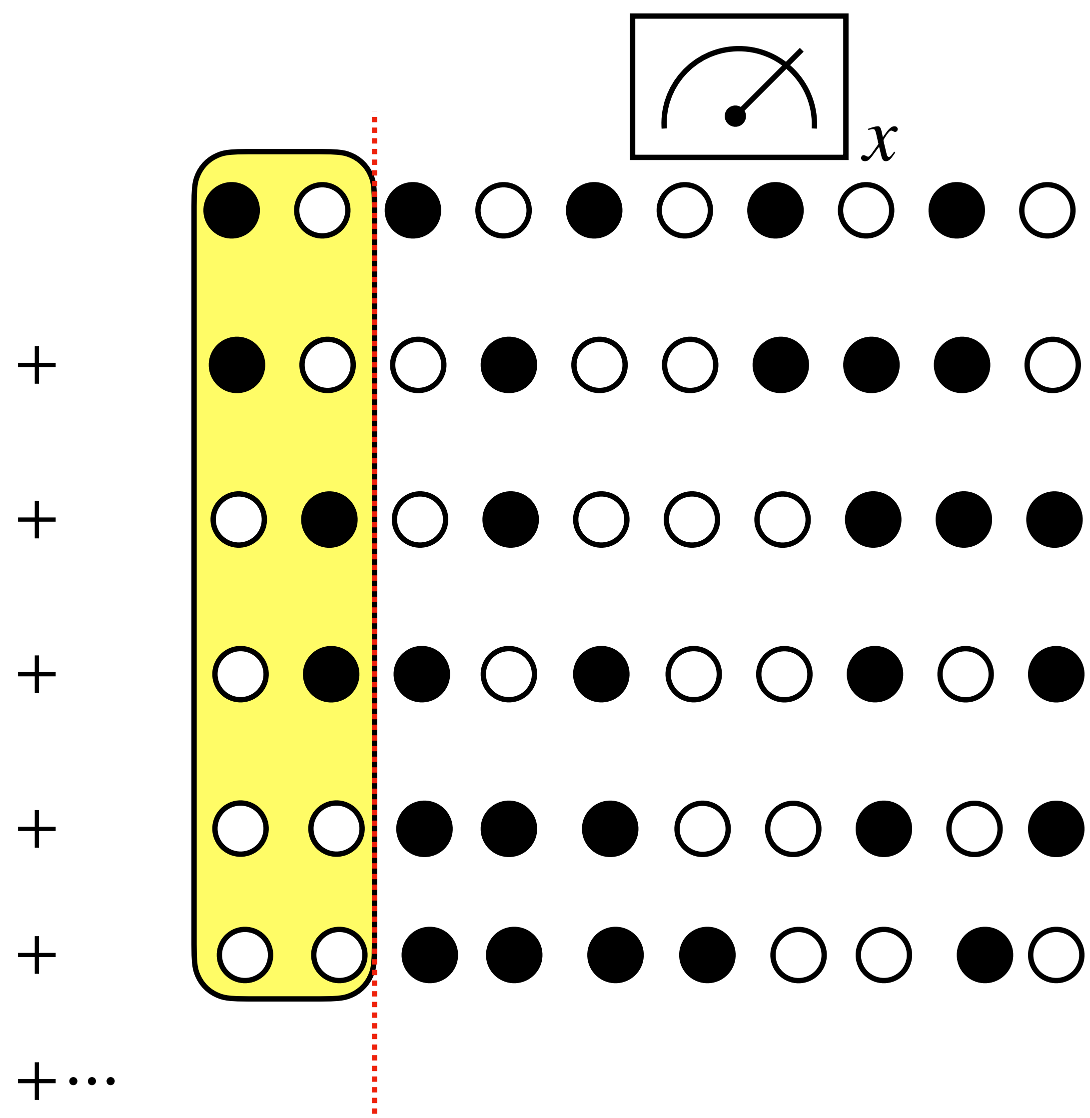

- Measure B in the computational basis x *"charge non-revealing basis"*
- Each bit-string x_B yields a state $|\psi_A(x_B)\rangle$ but we do not gain more information about its character

[Chang, Shrotriya, WWH, Ippoliti, 2408.15325]

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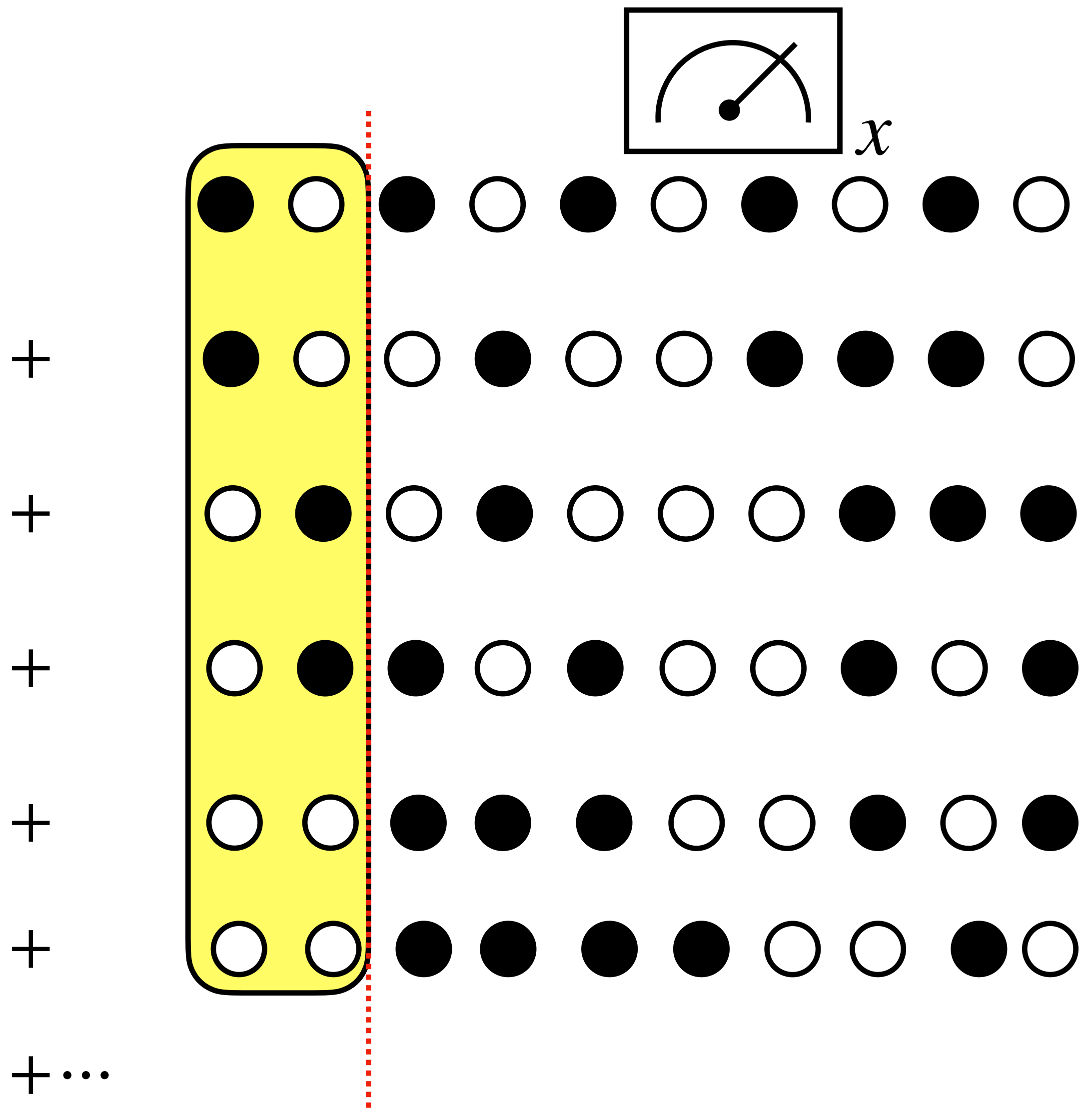

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$$\mathscr{E}_{PE} \to \mathscr{E}_{Scr,A}$$

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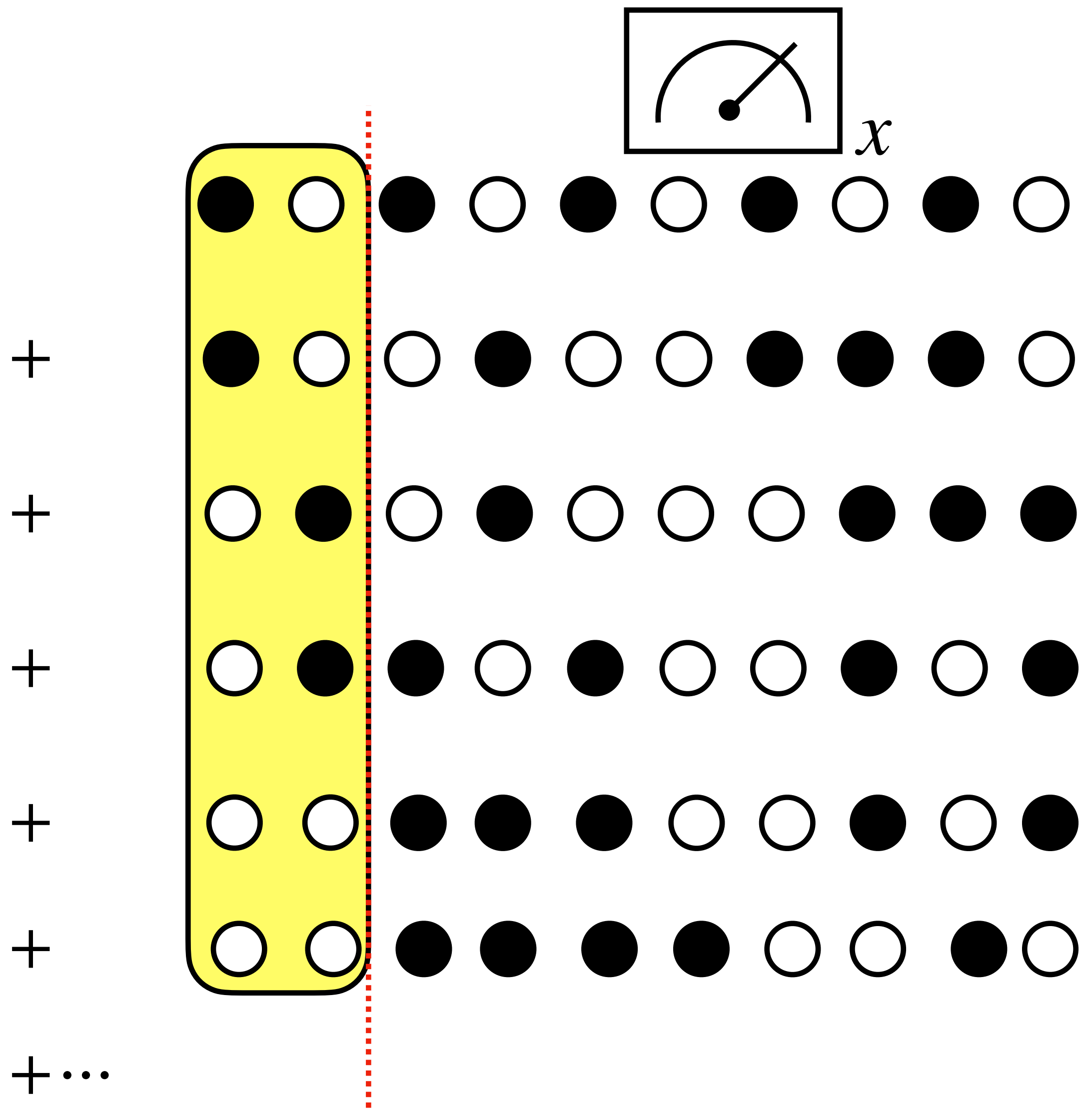

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Q: General theory incorporating charge fluctuations?

[Chang, Shrotriya, WWH, Ippoliti, 2408.15325]

Theory for charge revealing measurements

(Based on calculations using replica trick)

- Let the initial state have charge fluctuations $p(Q)$, i.e., $|\Psi\rangle = \sum_Q p(Q)|\Phi_Q\rangle$

[Chang, Shrotriya, WWH, Ippoliti, 2408.15325]

Theory for charge revealing measurements

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- Measure in a charge revealing basis (e.g. z), to get z_B which carries charge Q_B. This occurs with probability $\pi(Q_B) = \sum_Q \pi(Q_B|Q)p(Q)$

$$\pi(Q_B|Q) = \binom{N_A}{Q_A}\binom{N_B}{Q_B}\binom{N}{Q}^{-1}$$

[Chang, Shrotriya, WWH, Ippoliti, 2408.15325]

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- Knowledge of Q_B updates our belief of Q_A: $\pi(Q_A|Q_B) = \pi(Q_B|Q)p(Q)/\pi(Q_B)$

[Chang, Shrotriya, WWH, Ippoliti, 2408.15325]

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- Average state should be $\rho_A(Q_B) = \sum_{Q_A} \rho_{Haar,A}(Q_A)\pi(Q_A|Q_B)$ which unravels into a Scrooge distribution $\mathscr{E}_{Scr.}(\rho_A(Q_B))$

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- Limiting distribution will be a convex sum over Scrooges (generalized Scrooge):

$$\mathscr{E}_{PE} \to \sum_{Q_B} \pi(Q_B)\mathscr{E}_{Scr.}(\rho_A(Q_B))$$

Note: it is universal, depending only on $p(Q)$

[Chang, Shrotriya, WWH, Ippoliti, 2408.15325]

(Partial) Theory for charge non-revealing measurements

- Let the initial state have definite charge $p(Q) = \delta_{Q,Q_0}$

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- Let the initial state have definite charge $p(Q) = \delta_{Q,Q_0}$
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- This does not yield new information about charge on A, so prior and posterior probabilities are both $\pi(Q_A \mid Q_0) = \binom{N_A}{Q_A}\binom{N_B}{Q_0 - Q_A}\binom{N}{Q_0}^{-1}$

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- Limiting distribution will be single Scrooge (stingy unraveling of ρ_A)

$$\mathscr{E}_{PE} \to \mathscr{E}_{Scr.}(\rho_A)$$

Note when $Q_0 = N/2$, $\mathscr{E}_{Scr.} \to \mathscr{E}_{Haar}$